Chapter 1

Laser Cooling and Trapping of Atoms: A Brief Survey

1.1 Introduction

This chapter begins with the motivation for laser cooling of atoms, outlining possible uses of ultracold atoms. The theoretical background for laser cooling of atoms is then given, beginning with the interaction of a two level atom with light, the origin of the viscous drag and how an optical molasses may be formed. Doppler and sub-Doppler cooling are elaborated upon. Finally, the techniques of confining the cold atoms are discussed.

The technique of laser cooling enables one to reduce the temperature of a small collection of atoms to a few hundreds of micro-Kelvin or lower. Atoms cooled to sub-milli-Kelvin temperatures permit a variety of studies that are not possible with room temperature atoms. The reduced velocity of atoms at low temperatures opens the door to high resolution spectroscopy on such atoms since the absorption and emission linewidths of such atoms are limited only by the lifetime of the atom in the excited state and not by the Doppler broadening. Such high resolution spectroscopy is of interest per-se and also for testing the foundations of quantum mechanics. The reduced velocity of atoms and the consequent increased resolution of frequency measurement has resulted in a phenomenal increase in accuracy of the time standard.

Cold atoms have de-Broglie wavelengths comparable to the range of their mutual interaction. As the velocity is reduced, an atom can change its internal state while the scattering force is still prevalent. Novel effects of collisions among such atoms are expected. A differ-

ent theoretical formalism is necessary to describe such collisions.

If atoms obeying Bose statistics are cooled below the condensation temperature a macroscopic fraction of the total number of atoms will condense into the ground state. This is called Bose-Einstein condensation where the macroscopic collections of atoms is described by a single quantum mechanical wavefunction. The condensate exhibits novel properties. If, on the otherhand, atoms obeying Fermi-Dirac statistics are cooled to low temperatures, the gas of such atoms will become degenerate. One can study the behaviour of such degenerate Fermi gases.

Using photo- association one can form dimers of the atoms at low temperatures. One can study such cold molecules and even form Bose-condensates of these molecules.

The list of practical applications of cold atoms is quite lengthy. To mention just a couple of these, cold atoms may be used for nano-lithography and for high sensitivity gyroscopes.

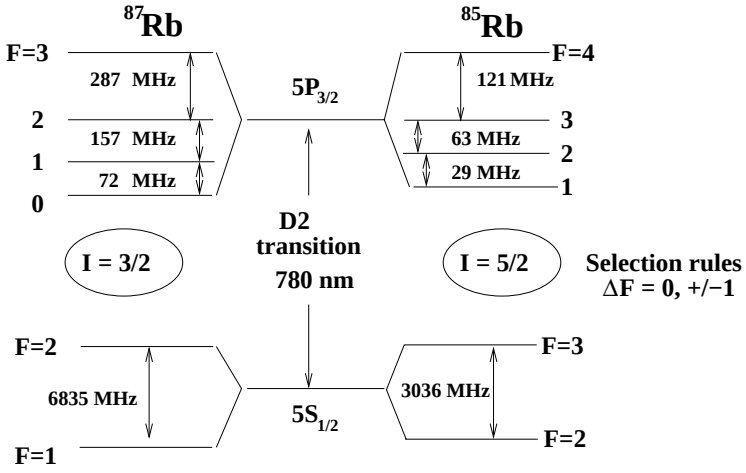

Figure 1.1: Energy level structure for ^{85}Rb and ^{87}Rb atoms. (Not to scale)

1.2 Energy Levels of Alkali Atoms

Most experiments on laser cooling, and the theoretical analyses have been on alkali atoms. These have a lone electron outside a closed shell, and thus have relatively simple energy level structures. In the usual notation the ground state of the atom is designated by $n^2S_{1/2}$, where n = 1,2,3.... is the principal quantum number. S indicates that the total orbital angular momentum L = 0. The value of the total electronic angular momentum J = 1/2. The 2 denotes a doublet, and is equal to 2S+1, where S is the total electronic spin angular momentum. Such an electron may be excited to either $n^2P_{1/2}$ or $n^2P_{3/2}$. These transitions are referred to as D1 and D2 transitions respectively. The corresponding absorption/emission lines differ in wavelengths by a few nanometers. (eg.780nm and 795nm for Rb, 589nm and 589.6nm for Na).

The nucleus of the atom has a spin angular momentum, **I**. The interactions between **J** the total electronic angular momentum and **I** the nuclear spin angular momentum, results in the total angular momentum quantum number **F** which varies in steps of unity from a maximum value $J + I$ to a minimum value $|J - I|$. The corresponding energy levels differ by small amounts, and so the interaction is called the hyperfine interaction. Different isotopes of an atom have slightly different energy values arising from the difference in their masses. We take the case of Rubidium which will be the subject of study in the present thesis. The most abundant isotope, ^{85}Rb has a nuclear spin **I** = 5/2. The ground state $^2S_{1/2}$ with J = 1/2, thus has two hyperfine sub-levels 5/2 ± 1/2 ie. F = 3,2. The excited atomic level $^2P_{3/2}$ splits into four levels F = 4,3,2,1. The ground state hyperfine levels differ in energy by approximately 3 GHz, while the excited state levels differ by a few tens to a few hundreds of MHz (see figure 1.1 and Tables 1.1, 1.2). The dipole selection rule allows transitions between states for which the F values differ by ±1 or zero. Thus from each ground state three hyperfine transitions are allowed giving rise to a triplet spectrum. Consequently, the lines in the spectrum are separated in frequency by a few tens to a few hundreds of MHz. The two sets of triplets arising from the two ground hyperfine states differ in frequency by approximately 3 GHz

(see table 1.1). Rubidium has another isotope ^{87}Rb which has a significant abundance, with nuclear spin I = 3/2. It has two ground hyperfine levels F = 1,0 and four excited hyperfine levels F = 3,2,1,0 (figure 1.1) . Once again this results in a triplet absorption spectrum. Thus in Rb vapour one sees four sets of triplet hyperfine absorption lines.

^{85}Rb (I= $5/2$)

Transition from F = 3 to	The transition frequency $\nu = \Delta E/h$
$F' = 4$	383998848.0 MHz
$F' = 3$	383998720.0 MHz
$F' = 2$	383998656.0 MHz
Transition from F = 2 to	The transition frequency $\nu = \Delta E/h$
$F' = 2$	384001760.0 MHz
$F' = 3$	384001696.0 MHz
$F' = 1$	384001664.0 MHz

^{87}Rb (I= $3/2$)

Transition from F = 2 to	The transition frequency $\nu = \Delta E/h$
$F' = 3$	383997600.0 MHz
$F' = 2$	383997344.0 MHz
$F' = 1$	383997184.0 MHz
Transition from F = 1 to	The transition frequency $\nu = \Delta E/h$
$F'= 2$	384004192.0 MHz
$F'= 1$	384004032.0 MHz
$F'= 0$	384004012.0 MHz

Table 1.1: Hyperfine transition frequencies of Rb atoms from F levels of $5S_{1/2}$ to the F' levels of $5P_{3/2}$, where the selection rule is $\Delta F = \pm 1, 0$ (Calculations were done using the formula in the reference [1]).

^{85}Rb (I= 5/2)

Energy level	shift $\Delta\nu$
$5S_{1/2}$, J = 1/2 , F = 3	1264.8875 MHz
$5S_{1/2}$, J = 1/2 , F = 2	-1770.8424 MHz
$5P_{1/2}$, J = 1/2 , F = 3	150.9000 MHz
$5P_{1/2}$, J = 1/2 , F = 2	-211.2600 MHz
$5P_{3/2}$, J = 3/2 , F = 4	100.2538 MHz
$5P_{3/2}$, J = 3/2 , F = 3	-20.4862 MHz
$5P_{3/2}$, J = 3/2 , F = 2	-83.8672 MHz
$5P_{3/2}$, J = 3/2, F = 1	-113.1813 MHz

^{87}Rb (I= 3/2)

Energy level	shift $\Delta\nu$
$5S_{1/2}$, J = 1/2 F = 2	2563.0051 MHz
$5S_{1/2}$, J = 1/2 F = 1	-4271.6753 MHz
$5P_{1/2}$, J = 1/2 F= 2	304.6500 MHz
$5P_{1/2}$, J = 1/2 F = 1	-507.7500 MHz
$5P_{3/2}$, J = 3/2 F = 3	194.0488 MHz
$5P_{3/2}$, J = 3/2 F = 2	-73.0762 MHz
$5P_{3/2}$, J = 3/2 F = 1	-230.1763 MHz
$5P_{3/2}$, J = 3/2 F = 0	-302.4313 MHz

Table 1.2: Hyperfine frequency shifts of Rb atoms w.r.t. the corresponding finestructure levels (Calculations were done using the formula in the reference [1]).

1.3 Slowing Down of An Atom

For simplicity we consider a two level atom with ground state $|g\rangle$ and excited state $|e\rangle$ separated by an energy $\Delta E = \hbar\omega_a$ and that transition is allowed between the two states by dipole selection rules. There are three basic radiative processes namely absorption, spontaneous emission and stimulated emission and these are depicted in figure 1.2.

Let a laser beam of angular frequency ω_L be incident on the atom. If ω_L is close to ω_a to within the Doppler width, the atom may make a transition from the ground state to the excited

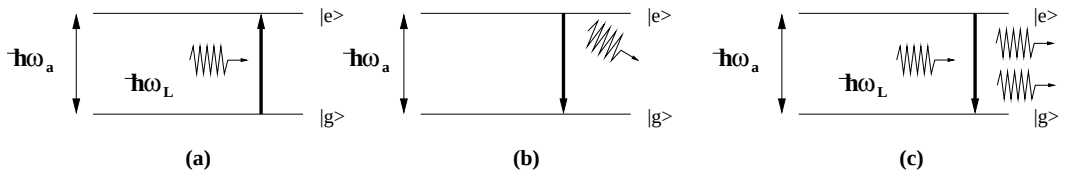

Figure 1.2: The three basic radiative processes (a) Absorption (b) Spontaneous emission (c) Stimulated emission. $\hbar\omega_a$ is the transition frequency in the rest frame of the atom, ω_L is the laser frequency, $|g>$ and $|e>$ are the ground and excited states respectively.

state. In addition to energy $E = \hbar\omega_L$, each photon carries momentum $\hbar\mathbf{k}$, where $\mathbf{k} = \frac{2\pi}{\lambda}\hat{k}$ the wave vector, and angular momentum $\hbar$. When an atom absorbs light, the additional energy is accomodated by going into an excited state and the additional momentum by recoiling. The angular momentum of the absorbed photon is accommodated in the form of the internal motion of the electrons. The converse applies for emission. The rate R_a at which the atom absorbs the photon depends on the intensity of the laser beam, the matrix element of the dipole moment of the transition and the detuning $\delta = \omega_L - \omega_a$ of the laser beam. If $\omega_L > \omega_a$ then δ is positive and the laser is said to be blue detuned, while if $\omega_L < \omega_a$ (δ negative) laser is said to be red detuned. An atom stays in the excited state for a short time τ and then spontaneously returns to the ground state, emitting a photon in an arbitrary direction, which, in general, is different from the direction of the incident radiation. The corresponding emission line has a natural line width $\Gamma = 2\pi/\tau$. There is also an induced emission process in which the atom is stimulated by an incident photon to make a transition from the excited state to the ground state. The emitted photon travels in the same direction as the stimulating photon. When a photon is absorbed the atom acquires an additional momentum $\hbar\mathbf{k}$, where $\mathbf{k}$ is the wave vector of the incident radiation. When the atom emits a photon with wave vector $\mathbf{k}'$ it acquires a recoil momentum $-\hbar\mathbf{k}'$. If a laser beam is incident on an atom in the ground state, and one photon from that causes an upward transition followed by a second photon from the same stimulating a downward transition, the net change in momentum is zero as $\mathbf{k} = \mathbf{k}'$. On the other hand since $\mathbf{k}'$ is arbitrary in spontaneous emission, over a large number,

n, of absorption-spontaneous emission cycles the net momentum absorbed by the atom is

$$\begin{aligned} \sum_{i=1}^{n} \hbar\mathbf{k} - \sum_{i=1}^{n} \hbar\mathbf{k}' &= n\hbar\mathbf{k} - 0 \\ &= n\hbar\mathbf{k} \end{aligned}$$

Such an atom will experience a force from the laser beam. The rate of spontaneous emission by the atom is given by

$$R_{sp} = \Gamma \frac{I/I_s}{[1 + 2(I/I_s) + 4(\delta/\Gamma)^2]}. \tag{1.1}$$

Here I is the intensity of the laser beam and I_s is the saturation intensity, defined as the intensity at which the Rabi frequency Ω is equal to the spontaneous transition rate Γ. The Rabi Frequency is defined by

$$\hbar\Omega = |\mathbf{E}\cdot < g|\mathbf{d}|e > |, \tag{1.2}$$

where **E** is the amplitude of the electric field associated with the laser field and $< g|\mathbf{d}|e >$ is the matrix element of the electric dipole moment operator **d** between the ground and excited states, $|g >$ and $|e >$.

The force experienced by the atom is

$$\mathbf{F} = (\hbar\mathbf{k})R_{sp}, \tag{1.3}$$

Since Γ is of the order of a few tens of a MHz, for small detuning δ (i.e.$\delta/\Gamma \approx 1$), this force can be two to three orders of magnitude larger than the force of gravity. When the atom moves against the direction of the light beam this force will slow down the atom. This is the basic idea in laser cooling of atom. Table (1.3) gives a few relevant numbers for laser cooling of ^{85}Rb atoms.

For an atom moving towards the light, the frequency of the latter appears blue-shifted by kv, where v is the velocity of the atom. The detuning in the rest frame of the atom will be

$$\delta = (\omega_L - \omega_a + kv). \tag{1.4}$$

So the force F will depend on the velocity of the atom and will vary as the atom slows down. Thus, though a laser beam with certain detuning may effectively slow down atoms moving

Most probable velocity of Rb atoms in the atom beam at 300 K	250m/s
Resonance wavelength λ_0	780nm
Recoil velocity from the photon($\hbar k/m$)	6mm/s
Lifetime of the excited state	30ns
No. of photons required to stop the atom	73,000
Stopping distance at half maximum decelaration	1.8m

Table 1.3: Few relevant numbers for ^{85}Rb atoms.

fast, its effectiveness decreases when the atom is slowed down due to the reduced Doppler shift in frequency of the light beam. In order to maintain the deceleration at its maximum value as the atom slows down, it is necessary either to increase the laser frequency continuously to compensate for the reduction in Doppler shift or to decrease ω_0 as the atom moves forward. The first process is called chirping the laser beam. For atoms which have resonant frequency in the red part of the electromagnetic spectrum, one may use a laser diode as the source of the slowing beam and it is easy to chirp the frequency of the laser. Typically, one needs to vary the frequency over a range of $\approx$ 20 GHz, and this is achieved by varying the current through the diode. For atoms like sodium, for which the resonance frequency is in the yellow, one will have to use a dye laser as the source, as suitable solid state lasers are not available at this wavelength. Chirping a dye laser is more complicated than chirping a diode laser. By using an electro-optic modulator one can obtain a higher frequency side band. The frequency of the side band differs from the frequency of the laser beam by the frequency of modulation. By using the side band laser beam as the slowing down beam, one can achieve chirping by continuously chirping the frequency of modulation. Ertmer et al [2] used such a technique to slow down a beam of sodium atoms. The more common technique is to change the transition frequency by using the Zeeman shift produced by a magnetic field. Using a coil tapering towards one end one may adjust a decreasing magnetic field seen by the atom as it moves forward. The field gradient is adjusted to keep the detuning of the laser in the rest frame of the moving atom constant as it moves forward with a progressively decreasing velocity. This technique, first used by Prodan et al [3] to slow down a beam of sodium atoms, is widely used for various elements.

1.4 Doppler Cooling of Atoms In Three Dimensions: The Optical Molasses

Hansch and Schawlow [4] suggested that a viscous drag may be produced on an atom moving in any direction by shining on the atom three pairs of counter-propagating beams derived from the same laser and travelling along three orthogonal directions (see figure 1.3). A similar effect was proposed by Wineland and Dehmelt [5] for trapped ions.

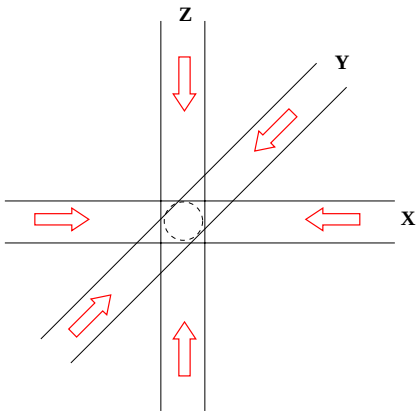

Figure 1.3: Configuration of three counter-propagating laser beams (derived from the same laser) to realize the optical molasses(dotted region).

Let us consider three pairs of counter-propagating beams along x,y, and z directions, with all of the same detuning, and same intensity. Consider an atom moving with velocity **v**, with velocity component v_x along **x**. The photon travelling in the -**x** direction will suffer a Doppler shift -kv_x, when seen from the atom's frame of reference. The force F_x on the atom in the +**x** direction is

$$F_x = (\hbar k)[R_{sp}(\delta_0 - kv_x) - R_{sp}(\delta_0 + kv_x)] \tag{1.5}$$

where

$$\delta_0 = \omega_L - \omega_a \tag{1.6}$$

and $R_{sp}(\delta')$ is the function defined in equation 1.1. If kv_x is small compared to δ_0 then we may stop with the first power in kv_x in a Taylor series expansion of R_{sp} and get.

$$F_x = (\hbar k)[(dR_{sp}/d\delta)_{\delta=\delta_0}(-2kv_x)] \tag{1.7}$$

$$= -\alpha v_x \tag{1.8}$$

where

$$\alpha = 16(\hbar k^2)(-\delta_0/\Gamma)\frac{I/I_s}{[1 + 2(I/I_s) + 4(\delta_0/\Gamma)^2]^2} \tag{1.9}$$

If δ_0 is negative(i.e. the laser beam is red detuned) then α is positive and the force F_x opposes the velocity component v_x and reduces the velocity. In fact for each velocity component v_i, the corresponding force component F_i can be written as

$$F_i = -\alpha v_i \tag{1.10}$$

All velocity components of the atom experience a retarding force and thus the velocity of the atom decreases. The overlapping region of the three laser beams acts as a viscous medium slowing down the atom. Such a configuration of red detuned laser beams is therefore called an optical molasses.

Spontaneous emission, on the other hand, tends to increase the random kinetic energy of the atom. Since the recoil momentum of the atom is of the same magnitude ($\hbar k$) but random in direction, the increase in the square of the momentum in a time t can be written as

$$\Delta < p^2 >= 2Dt \tag{1.11}$$

where the diffusion coefficient

$$D = 6(\hbar k)^2 R_{sp}(\delta_0) \tag{1.12}$$

taking into account that there are six overlapping beams, each having a spontaneous transition rate $R_{sp}(\delta_0)$. Spontaneous emission produces heating. When the rate of cooling αv^2

equals the rate of heating D/m a steady state will be obtained. So the steady state velocity v is given by

$$v^2 = D/(\alpha m) \tag{1.13}$$

Equating mv^2 to $3k_BT_f$ where k_B is the Boltzmann constant and T_f is the final temperature we get

$$T_f = D/3k_B\alpha \tag{1.14}$$

$$= (\hbar\Gamma/k_B)[1 + 2(I/I_s) + 4(\delta_0/\Gamma)^2]/[8(-\delta_0/\Gamma)]. \tag{1.15}$$

If the intensity ratio I/I_s becomes small compared to unity, it can be neglected in the numerator. The minimum value of T_f is obtained when $(-\delta_0/\Gamma) = 1/2$. The minimum value of the temperature is

$$T_{fmin} = (\hbar\Gamma/2k_B) = (h/2k_B\tau). \tag{1.16}$$

This is the simple theory of Doppler cooling.

This theory leads to two consequences:

1. For low intensities both D and α are proportional to the intensity of the laser beam, and thus the final temperature is independent of the intensity of the laser beam.

2. The lowest temperature will be attained when the laser is red detuned to δ_0 = - $\Gamma/2$, and this gives the Doppler limit to cooling as (h/$2k_B\tau$).

The minimum temperature achievable by the Doppler cooling is $240\mu K$ for Na, 160 μK for Rb and $125\mu K$ for Cs.

The atoms follow a zig-zag path through the molasses due to absorption-spontaneous emission cycles. The confinement time of atoms in the optical molasses can be estimated. For sodium this confinement time is about 1s if the molasses region has a diameter of 1 cm. On the other hand in the absence of the light beams the atom with the terminal velocity will take about 20 ms to cover this distance.

The first experimental study of these predictions of the Doppler cooling in the optical molasses was carried out in the Bell Telephone Laboratories by Steven Chu and his collaborators [6]. The experiment was performed in a UHV chamber with six anti-reflection coated optical windows. A beam of sodium atoms was produced by shining 10ns pulses of 32 mJ energy from a Nd:YAG laser on a pellet of sodium, which produced sodium ions and neutral sodium atoms. The majority of the neutral atoms had an energy of 3.5eV. There was, however, a small fraction of atoms at a mean temperature of 1000K. At this temperature the mean velocity of the atomic beam was 2×10^4cm/s.

The molasses could effectively slow down atoms with an initial velocity of 3000 cm/s or less. So the beam was slowed down to 2×10^3 cm/s in 0.5 ms by a red detuned laser beam turned to the D2 hyperfine transition from F=2 to F'=3. The slowing down beam was then shut off. The atoms entered the optical molasses region where they resided for about 100 ms. The six molasses beams were derived from a single cw stabilized ring dye laser red detuned by - $\Gamma/2$. The laser beam could also raise atoms from the F = 2 to $F' = 2$ transition. About 1 in 1000 atoms would be raised to the level F'=2. These atoms would come down to the hyperfine levels F=2 and F=1 which are separated by 1.7 GHz. The atoms which came to the ground state F = 1 would go out of the cooling cycle. So a second repumper laser tuned to the transition from F=1 to $F' = 2$ was used to bring the atoms back into the cooling cycle. This beam was derived from the cooling laser by using an electro-optic modulator at 1.7 GHz. The above experiment was described in some detail as it typifies the arrangement one should use for laser cooling experiment with any atoms. For any alkali atom one has to use a repumper laser to bring the atoms transferred to the dark ground state back into the cooling cycle. With Rb and Cs even at moderately high temperatures of a few hundred degrees, there is a sufficient fraction of atoms with velocities less than the capture velocity. It is therefore not necessary with these heavy atoms to use a Zeeman slower.

With the above arrangement Chu et al performed two experiments: one to determine the spatial diffusion coefficient for the atoms in the molasses and the other to determine the final temperature of the atoms in the optical molasses. For the diffusion coefficient the fluorescent

intensity of the radiation scattered by the atoms in the molasses region was measured as a function of time. This intensity is proportional to the number of atoms in the molasses region. By fitting it to the theoretical expression for the number of atoms as a function of time from diffusion theory one can obtain the diffusion coefficient. One parameter, namely the radius R of the molasses region, should be known for the evaluation of the diffusion coefficient. This was chosen as the distance from the centre to the point in the molasses where the fluorescent intensity was 10% of its maximum value.

The temperature was determined by the release and re-capture method, where the molasses beams were shut off for a time τ and switched on again. The fluorescent intensity in the molasses was measured as a function of time. In a time τ atoms with a velocity v>2R/τ will leave the molasses region. Changing τ changes this limiting velocity. One can fit experimental data of the variation of fluorescent intensity as a function of τ with a theoretical calculation based on a Maxwellian distribution of velocities at a temperature T. The calculation is not straight forward and the temperature value obtained by such a fit will be influenced by the uncertainties in the radius R of the optical molasses region. The authors estimated the final temperature of the cloud of Na atoms in the molasses to be 240μK with an upper bound of 440 μK and a lower bound of 180μK. Their results appeared to support the simple Doppler cooling theory.

However, more careful experiments by P.D. Lett et al.[7] were performed on Na atoms in optical molasses. They shielded the molasses region so that the magnetic field in that region did not exceed 0.02mT, and changed the laser detuning from 0 to -4Γ. They measured the velocity distribution of the cold atoms by four different methods: a) by measuring the time of flight of the atoms as they fall down under gravity to reach a probe beam kept 1 to 1.9 cm below the molasses region; b) by locating the probe beam at a height h above the molasses region and measuring its fluorescent intensity; one measures how many atoms in the molasses region will have a vertical velocity greater than $\sqrt{2}$gh; c) by measuring the distribution of the horizontal velocity component and d) by release and recapture method used by Chu et al.

For a detuning of -2Γ these four methods yielded a temperature of 40μK for the cold atoms which was far below the minimum temperature attainable by Doppler cooling. When the temperature was measured as a function of the red detuning of the laser beam the minimum temperature was obtained at a detuning $\delta = -4\Gamma$. This is in contradiction to the theoretical result for Doppler cooling. These results were later confirmed by a group in Stanford and another in Ecole Normale Superieure in Paris.

A clue was provided for this additional cooling by the experiments of Lett et al. They noted that a) increasing the magnetic field increased the temperature of the cloud of atoms and b) changing the state of polarisation of one pair of molasses beams also increased the temperature.

In the two level atom we considered the levels to be singly degenerate. An actual atomic level consists of several Zeeman sublevels m_J = -J, -(J-1),.....,(J-1),J. The fact that a magnetic field has an effect on the temperature indicated that the Zeeman sublevels of the ground and excited states have a role to play in determining the temperature.

In optical molasses the superposition of laser beams can cause a change in the polarization of the resultant electric vector over a distance of half the wave-length. The second observation indicates that the polarisation gradient has a role to play.

1.5 Mechanism For Sub-Doppler Cooling

The mechanism of sub-Doppler cooling depends on (a) light shift and (b) optical pumping. Light shift is the shift in the energy levels of the atom produced by its interaction with the near-resonant laser beam. Different Zeeman sub-levels are shifted by

$$\triangle(m_J) = C^2\Omega^2/\delta \tag{1.17}$$

Ω is the Rabi frequency of the laser beam, C is the Clebsch-Gordon coefficient for the transition from J, m_J, to J', m'_J induced by the state of polarisation of the light at that point. σ^+ polarised light can only induce a transition from m_J to $m_{J'} = m_J + 1$, and σ^- polarised light from m_J to $m_{J'} = m_J - 1$. The corresponding Rabi frequencies will be different. If the ground

state corresponds to $J = 1/2$, the light shift caused to the ground sub-level $m_J = 1/2$ by light of σ^+ polarisation will be three times as large as the shift of the ground sub-level $m_J = -1/2$. The light shift is a second order Stark effect and is much smaller in magnitude than the shift of the sub-levels produced by a magnetic field.

The probability for a transition depends on the magnetic quantum numbers of the ground and excited sub-levels. Consider a ground level J = 1/2 and an excited level J'= 3/2. There are two sub-levels for the ground level with $m_J = 1/2$ and -1/2 and four excited sub-levels $m_J = 3/2, 1/2, -1/2, -3/2$. If the light is σ^+ polarised it will raise the atoms from $m_J = 1/2$ to $m_{J'} = 3/2$ and $m_J = -1/2$ to $m_{J'} = 1/2$. The atoms raised to the state $m_J = 1/2$ can come down by spontaneous transition to $m_J = 1/2$ and $-1/2$, the first with a probability twice as large as the second. So, after many absorption emission cycles all the atoms in the ground state will be in the state $m_J = +1/2$, completely depopulating the state $m_J = -1/2$. This is called optical pumping. Similarly in the presence of σ^- all the atoms will be pumped to the ground state $m_J = -1/2$. When the light is linearly polarised both the ground sub-levels will be equally populated.

1.5.1 Lin $\perp$ Lin configuration:

Linearly polarized light, polarized in orthogonal direction, counter-propagate along z (figure 1.4). The resultant polarization will vary from point to point. Let the light be linearly polarised at the point A. At point B to the right of A and at a distance $\lambda/8$ from A, the two light waves will have a phase difference $\pi/2$ between them and the resultant polarisation will be say σ^+. Consider the points C,D, and A' each point separated from its preceeding point by $\lambda/8$. The light will be linearly polarised (at right angles to the polarization at A) at C, will be σ^+ polarized at D, and will be again linearly polarised(with the same state of polarization as at A) at A'. Thus the state of polarization will recur as shown in figure 1.4. Let us consider an atom moving from A to the right. At B the ground Zeeman sub-level $m_J = 1/2$ will be shifted down relative to the ground Zeeman sub-level $m_J = -1/2$ because of the light shift. At C the Zeeman sub-lebels will be light shifted by the same energy and at D the Zeeman

sub-level $m_J = -1/2$ will have a lower energy. As shown in figure 1.5 the energy of the sub-levels $m_J = 1/2$ and $m_J = -1/2$ will have a periodic variation in the light shifts with a phase difference of π.

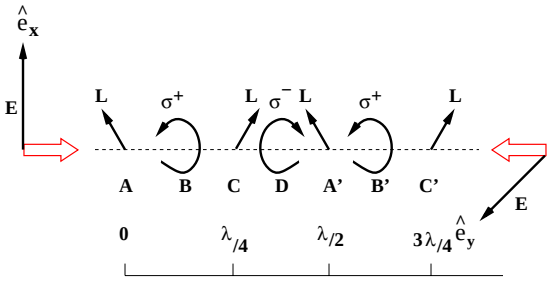

Figure 1.4: Lin ⊥ Lin Configuration. State of polarization of light (L, σ^+, L', σ^-, L, σ^+, L' at points A,B, C, D, A′, B′ and C′)

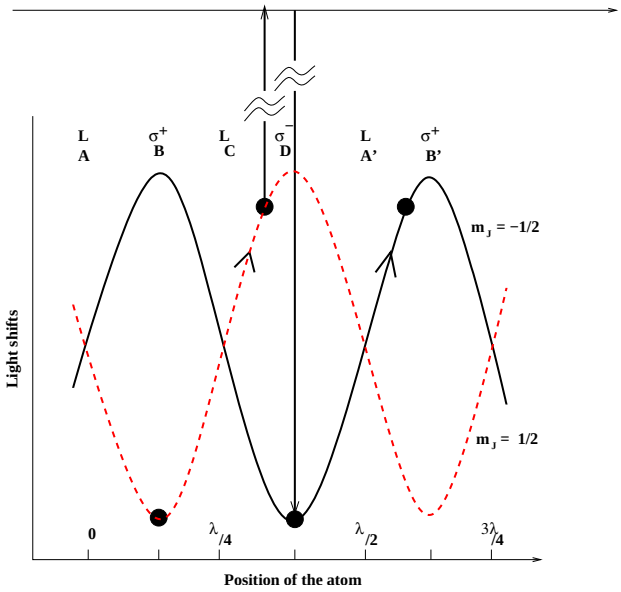

Figure 1.5: Variation of light shift of the ground Zeeman sub-lebels as a function of position of the atom to explain Sisyphus cooling.

Let us start at B, with all the atoms pumped to the sub-level $m_J = 1/2$ which has the lowest energy. Let the atoms move to the right and reach the point D in a time t equal to the optical pumping time τ_{op}. At D part of the kinetic energy of the atoms at B has been

converted into potential energy. But at D the atoms will all suffer optical pumping to the state $m_J = -1/2$ which will now have the minimum energy same as at B. The excess potential energy at D is carried away by the photons in the optical pumping process. The atoms will start with a smaller kinetic energy than at B and will climb the potential as they move from D to B′. Again at B′ they suffer optical pumping to the state $m_J = 1/2$ which has the lowest potential energy. Thus at each pont D, B′, D′.... the atoms lose kinetic energy and so will cool. However it is only atoms with velocity approximately equal to λ/τ_{op} that suffer cooling in this process. This process is called Sisyphus cooling.

The rate at which energy is lost is given by

$$dW/dt = -(\hbar\triangle')/\tau_{OP} \tag{1.18}$$

Equating this to -$\alpha_{Sis}v^2$, we get for frictional coefficient.

$$\alpha_{Sis} = (\hbar\triangle')/\tau_{OP}v^2 \tag{1.19}$$

and using in this for v the value λ/τ_{OP}, and substituting for $1/\tau_{OP}$ the value $\Omega^2\Gamma/\delta^2$ and for the light shift the value $\triangle' = \Omega^2/|\delta|$, we may rewrite equation 1.19 as

$$\alpha = (\hbar k^2|\delta|/\Gamma) \tag{1.20}$$

The damping coefficient in the Sisyphus cooling mechanism

1. is independent of the intensity since the light shift Δ' and the reciprocal of the optical pumping life time are proportional to the intensity

and

2. increases as the detuning δ increases.

The diffusion coefficient D for heating is the same as in Doppler cooling and is proportional to the intensity. The steady state temperature is then

$$k_B T_f = C(\hbar\Omega^2/|\delta|) \tag{1.21}$$

The final temperature will be lower, the lower the intensity of the laser beams and the farther the beams are red detuned. In fact the final temperature must vary linearly with the ratio of the intensity I of the laser beam to the detuning $|\delta|$.

A more rigorous calculation of the frictional force carried out by Dalibard and Cohen-Tannoudji [8] shows that the force F varies as $v/[1 + (v/v_c)^2]$, where the critical velocity $v_c = \lambda/\tau_{OP}$. When v<< v_c the drag force is proportional to v while for the case v>> v_c the drag will be proportional to 1/v. It may therefore, appear that Sisyphus cooling will only be effective for v<< v_c. However, in Sisyphus cooling there is an additional source of heating arising from the fluctuations in the light shift gradient due to fluctuations in the light intensity. This heating also varies as 1/v for v>> v_c. So, Sisyphus cooling is also effective for velocities v>> v_c.

1.5.2 $\sigma^- - \sigma^+$ Configuration:

A second configuration for the cooling beams is $\sigma^+ - \sigma^-$. This configuration is shown in figure 1.6.

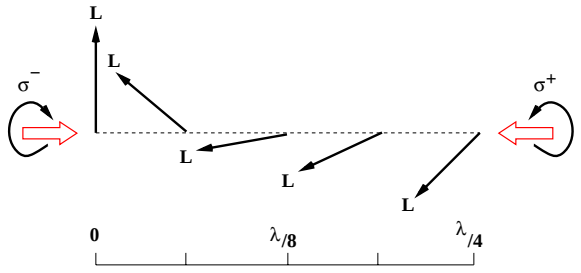

Figure 1.6: σ^- - σ^+ Configuration of the laser beams.

In this configuration the light is linearly polarized everywhere. But the direction of linear polarization rotates through an angle ϕ as one moves a distance z given by

$$\phi = 2kz \tag{1.22}$$

This case is more involved to treat theoretically. It is sufficient to say that in the presence of laser beams in this configuration, the atoms are polarised i.e. all the Zeeman ground sub-levels are not equally populated. This polarization occurs only for atoms with J>1/2. Because of the difference in population of the ground sub-levels more photons are absorbed from the laser beam of one polarisation than from the laser beam of another polarisation and this gives rise to a drag force proportional to the velocity. That is why this cooling is called Motional Orientational Cooling. Calculations indicate that the drag coefficient α for this configuration is less by a factor $\Gamma/|\delta|$ than the drag coefficient α_{Sis}. In Sisyphus cooling there is a second contribution to heating arising from fluctuations in the light shift gradient due to fluctuations in the intensity of the beams, in adition to spontaneous emission. In motional orientation cooling this contribution is absent as there is no gradient in the light shift. So both Sisyphus cooling and motional oreintation cooling are equally effective in producing sub- Doppler cooling.

1.6 Experimental Verification

Salomon et al. [9] performed the following experiment. Since the hyperfine level structure of Cs was better suited for the experiment a beam of Cs atoms was used. This beam issuing from an oven was slowed down using a counter-propagating chirped frequency laser beam. The slowed atoms wandered into the molasses region. The molasses beams were red detuned below the transition $6S_{1/2}$ F = 4 to $6P_{3/2}$ F′= 5. The cooling beams were obtained from a stabilised ring dye laser. Each of the molasses beams had a maximum intensity of 4mW/cm^2. The intensity could be decreased to 0.04mW/cm^2. The laser detuning was adjustable from zero to -48Γ. The molasses region was magnetically shielded so that the magnetic field did not exceed $1\mu T$.

Two polarization configurations were used (a) lin || lin and (b) lin $\perp$ lin. In the first configuration the counter-propagating beams along any one direction were parallel, but the polarization along the three mutually orthogonal directions were orthogonal.

About 10^8 atoms were collected in the molasses in 1.5s. The slowing laser beams were

turned off. The detuning of the molasses laser beams were rapidly switched on and the intensity was lowered. A waiting time of 100ms was given for the atoms to reach a constant temperature. The molasses beams were turned off and the atoms were allowed to fall under gravity a distance of 70 mm to reach the probe region. Here fluorescence from a rectangular paralleopiped of height 2mm, width 7mm and length 16 mm was imaged onto a photo detector. The fluorescence was produced by a probe beam turned to $6S_{1/2}$ F = 4 to $6P_{3/2}F' = 5$. Each atom absorbed 10^4 photons. To reduce the error in the velocity determination by this time of flight technique from the size of the cloud, a short pulse of laser light was made to fall for 7ms horizontally on the cloud. This pulse had a central shadow of 2mm width cast on the cloud. Atoms outside this width would get a transverse velocity by absorbing the light. During their free fall to the probe region they would travel also horizontally and would enter the probe beam outside the rectangular parallellopiped being imaged. Only atoms in the shadow region would enter the probe beam within the rectangular parallelopiped being imaged and only the fluorescence of these atoms was recorded as a function of time. Figure 1.7 shows one such time of flight recording. The fluorescence intensity is plotted as a function of the time of transit. The distribution was shown to result from a Maxwellian distribution of the velocity by comparing the widths of the curve at different values of the intensity of the fluorescence signal as a function of the maximum intensity. There was very good agreement between the measured and calculated widths showing that temperature equilibrium was attained within 100ms. The measured temperature was $2.5 \pm 0.6\mu$K, while the Doppler cooling limit is 125μK.

For a given detuning of the molasses beams it was verified that the temperature was a linear function of intensity as predicted by theory. For different combinations of intensity and detuning the temperature was plotted as a function of $I/|\delta|$. Below 30μK the points lay on a straight line. The constant C in equation 1.21 was found to be 0.4. This experiment has been described in some detail to illustrate how the temperature of the cloud of atoms can be measured accurately and how such measurement substantiate the sub-Doppler cooling theory due to polarisation gradient.

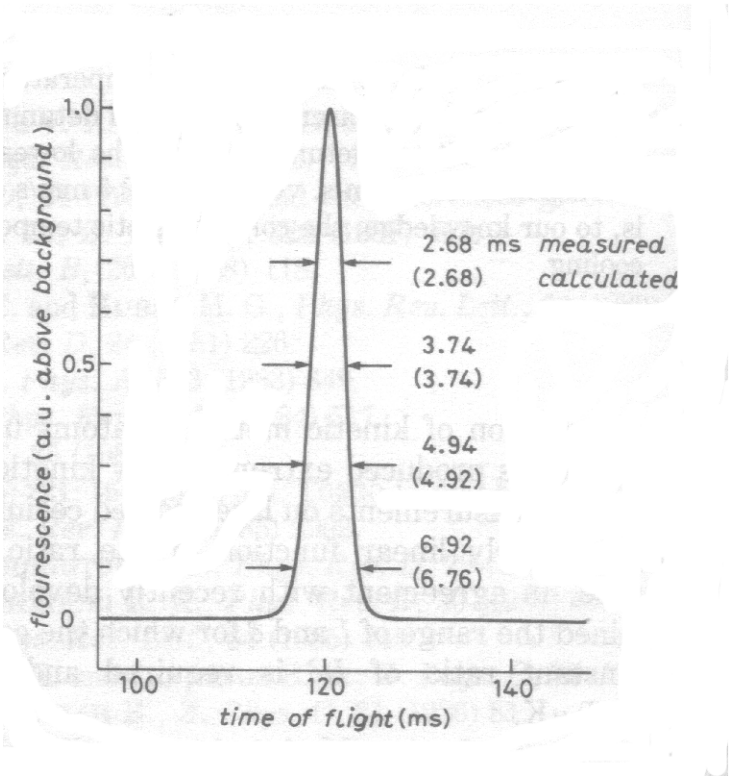

Figure 1.7: Experimental Time -of -Flight signal. (Figure taken from reference [9]).

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1.7 Trapping of cold atoms

The atoms stay in the molasses region for a longer period as they are slowed, and follow a zig-zag path. However, collisions with background hot atoms or the walls of the container cause heating. Therefore, they get out of the molasses region. The density of atom in the molasses region is also not very high. For carrying out experiments requiring several seconds it is necessary to trap the atoms and increase their density. Optical molasses can only damp the atomic velocity. It cannot trap the atoms, because the Doppler cooling force is only velocity dependent; it does not depend upon the position of the atom. To obtain trapping of the atoms a position dependent force is needed. There are essentially three different types of traps : (a) the magnetic, (b) the optical and (c) the magneto-optical. Of these we will discuss

in some detail the magneto- optical-trap making only a brief mention of the magnetic and optical traps.

1.7.1 Magnetic Trap:

For charged ions one can devise traps using suitable configuration of the electrostatic potential. For neutral atoms one can not devise such traps. One should make use of the weak magnetic moments of the atoms and devise suitable magnetic field configuration to create a potential well in which the atoms may be trapped. It is possible to have different configurations of coils carrying current to produce potential wells of desired shape and these are discussed by Bergmann et al.[11]. They are the quadrupole trap, the spherical hexapole trap and the Ioffe-Prichard trap. Of these we will describe the quadrupole trap as the coil configuration for this trap is also used in the magneto-optical trap.

Two identical coaxial coils are placed symmetrically about the origin and carry current flowing in opposite directions. The magnetic field is zero at the center and increases linearly in magnitude as one moves away from the center. The magnetic field is axially symmetric, the field gradient along the axis being twice as large in magnitude as the field gradient perpendicular to the axis. If we have an atom with J = 1/2, the Zeeman energy of one of the sub-levels m_J = -1/2 decreases as the distance increases from the origin while for the sub-level m_J = 1/2 the energy increases as the distance from the origin increases. Atoms in the state m_J = 1/2 will be trapped in the conical potential region while the atoms with m_J = -1/2 will leak out. This is the principle of the magnetic trap. In such a quadrupole trap the confining potential at a distance ℓ from the center in a given direction is

$$U(\ell) = \mu\beta\ell \tag{1.23}$$

Here μ is the magnetic moment, β is the field gradient in that direction. This is discussed in more detail shortly in the section on MOT.

The quadrupole magnetic trap was first used by Migdall et al [10] to trap sodium atoms in the state $3S_{1/2}(m_F = 2)$. The magnetic moment of Na in this state is nearly a Bohr magneton. At a location at which the magnetic field was 2T, the barrier height was 1.3 K, sufficient to

trap the sodium atoms with a velocity less than 30m/s. Two coaxial coils each of mean radius 2.7 cm, carrying 1900 A, were used. The coils were separated by 3.4 cm. The maximum usable field was only 0.025T giving a barrier height of 1.7 mK which could trap Na atoms with a velocity less than 3.5 m/s. The volume of the trap was 20 cc.

Of all the magnetic coil configurations the quadrupole trap produces the tightest confinement. However the centre of the trap, where the field is zero, corresponds to a "hole". The Larmor precession frequency close to the hole becomes smaller than the rate at which the moving atom sees the change in the field. Under these conditions spin-flip can occur and the atom after spin flip will leave the trap. One can show that the life time of the atoms in the trapped cloud is proportional to r^2 where r gives the characteristic linear dimension of the size of the cloud. As the atoms become cooler the size of the cloud decreases and the life time of the atom decreases.

1.7.2 Optical Trap

Several types of optical traps were proposed (Ashkin et al. [12] - [15]). These suffer from the disadvantages that the potential wells are shallow (depths $\approx$ 0.1 to 0.01K), and the volume of the trap is small. Further, atoms once trapped get heated by the random fluctuation of the light forces and will escape out of the trap. The trap that will be discussed at some length is the simplest one, namely the single beam gradient trap proposed by Ashkin in 1978 [14], and was realised in practice by Chu et al.

When an intense laser beam falls on an atom, the atom develops an induced dipole moment ($\mathbf{p} = \alpha\mathbf{E}$) due to the electric field of the laser light. The energy of the induced dipole is -1/2 αE^2. If the laser light intensity is non-uniform there is a force acting on the induced dipole given by

$$\mathbf{F} = -\nabla(-1/2\alpha E^2) \tag{1.24}$$

If the laser beam is red detuned, α is positive and the atom experiences a force pushing it to the region of maximum intensity. If one focuses an intense laser beam which is red detuned then the atom will be pushed towards the focal spot. This force called the dipole force

is different from the scattering force which was discussed in laser cooling. The latter arises from the absorption and spontaneous emission of photons and is related to the imaginary part of the dielectric constant. It is proportional to the intensity (for I<< I_s) and is a maximum when the laser is red detuned to half the natural linewidth. The dipole force arises from the real part of the dielectric constant and is present only if there is a spatial variation of the intensity of the laser beam. Even though the red detuning of the trapping beam may be more than the natural linewidth, the dipole force exists.

In an arrangement used by Chu et al. a push beam of Na atoms is created by shining 10μs pulse of a YAG laser on a pellet of Na. Such pulsed beams are produced in an interval of 0.1 to 10s. The atoms which have an initial velocity of 2×10^4 cm/s are slowed down to a velocity of 2×10^3cm/s by a counter-propagating frequency-chirped laser beam. The slowed atoms drift into an optical molasses created by three pairs of counter-propagating laser beams red detuned to $\Gamma/2$. These atoms are cooled to about 240μK. The density in the optical molasses is 10^6/cc and the confinement time is 0.5s.

This beam of sodium atoms is used to load the trap. The atoms wander into the trap by a random walk in the optical molasses. The reasonably long storage time in the molasses facilitates this.

The dipole trap is produced by a single strongly focused Gaussian laser beam red detuned to about 100 to 10,000. This beam is obtained from a dye laser and enters nearly parallel to one of the optical molasses beams. The power of the tapping beam is 220mW and the radius of focal spot is 10 m. The dipole force arising from the gradient of the intensity ensures radial stability of the trapped atoms. The dipole force can be derived from a conservative potential.

$$U = (\hbar\delta/2)\ln(1+p) \tag{1.25}$$

where,

$$p = (I/I_S)(\Gamma^2/4)[1/(\delta^2+\Gamma^2/4)] \tag{1.26}$$

p is the saturation factor. For small values of the detuning relative to Γ the potential is asymmetric in the axial direction. This arises due to the scattering force of the trapping

beams. However when the detuning is much more than Γ the scattering force becomes small compared to the dipole force and the asymmetry decreases. Calculations indicate a maximum depth of the potential well for a detuning of 650 GHz.

Chu et al. could capture about 500 atoms in a volume of 10^{-9} cc and found the life time in the trap to be longer than the residence time in the molasses.

The advantage of a single beam optical trap relative to the other optical traps are the following:

a. There are no standing waves in the single beam trap. Therefore heating of the atoms due to fluctuations in the dipole force is a mimimum.

b. The potential wells obtained are deeper than the any other optical trap. The wells are localised within a few wavelengths.

c. This type of optical trap is more suitable for achieving high atomic densities than other types of optical traps.

Unlike the magnetic trap, atoms in all the Zeeman sub-levels of the ground state are trapped in the optical trap. The optical trap has now been used to obtain Bose-Einstein condensate.

With beams of uniform intensity it is impossible to realize an optical trap because the divergence of the light field (electro-magnetic) is zero (Optical Earnshaw theorem). Optical traps have been realized using beams of variable intensity.

1.7.3 Magneto-Optical Trap

To have trapping of cold atoms, the Zeeman effect is used. In an inhomogeneous magnetic field both optical pumping and radiative cooling take place due to Zeeman effect and radiative selection rules. The entire setup is called Magneto-Optical-Trap MOT was first proposed by Dalibard in 1987, and the idea was tested by Pritchard and Chu [16] and worked at the first attempt.

The MOT is a very robust trap that does not depend on precise balancing of the counter-

propagating laser beams or on a very high degree of polarization.

1.7.3.1 One dimensional model

To understand the principle of a MOT the simplest 1-D model is chosen. The optical part of a 1D MOT is a pair of opposite circularly polarized red detuned laser beams that propagate along the z axis. The atoms are exposed to an anisotropic magnetic field produced by a pair of anti-Helmholtz field coils. We assume here the simple scheme, an atom with a $\mathbf{J}_g = 0 \rightarrow \mathbf{J}_e = 1$ transition where **J** is the angular momentum quantum number.

The ground state has only one Zeeman sublevel $m_g = 0$, but the excited state $\mathbf{J} = \mathbf{1}$, has three sublevels $m_e = +1, 0, -1$ which are degenerate for $\mathbf{B} = \mathbf{0}$. From the ground state the atom can be excited to one of these states, depending upon the polarization of the light. The frequency shift of its upper state is given by $\triangle\omega = \pm\mu B/\hbar$.

Suppose, the inhomogeneous magnetic field of the MOT has a constant gradient β and the zero point of the field is at the center of the MOT. The energy levels shift in different directions depending on which "side" of the center of the trap the atom is located (see figure 1.8). The energy shift is larger, the farther one moves away from the center, and the shift is given by

$$\triangle E = \pm\mu\beta z, \tag{1.27}$$

the + and - sign referring to $m_J = \pm 1$ levels respectively. μ is the magnetic moment of the atom. If the laser is tuned below the $\mathbf{B} = \mathbf{0}$ resonance frequency, the atom at z' will absorb more σ^- photons than σ^+ photons and consequently will feel a net time-averaged force towards z = 0 where the magnetic field is zero. On the other side of the center of the trap, the roles of the $m_g = \pm 1$ states are reversed and now more light is absorbed from the σ^+ beam, again driving the atom towards the center.

This situation is analogous to the velocity damping in an optical molasses from the Doppler effect as discussed earlier, but here the effect operates on the position space whereas for molasses it operates on velocity. Since the laser light is detuned below the atomic res-

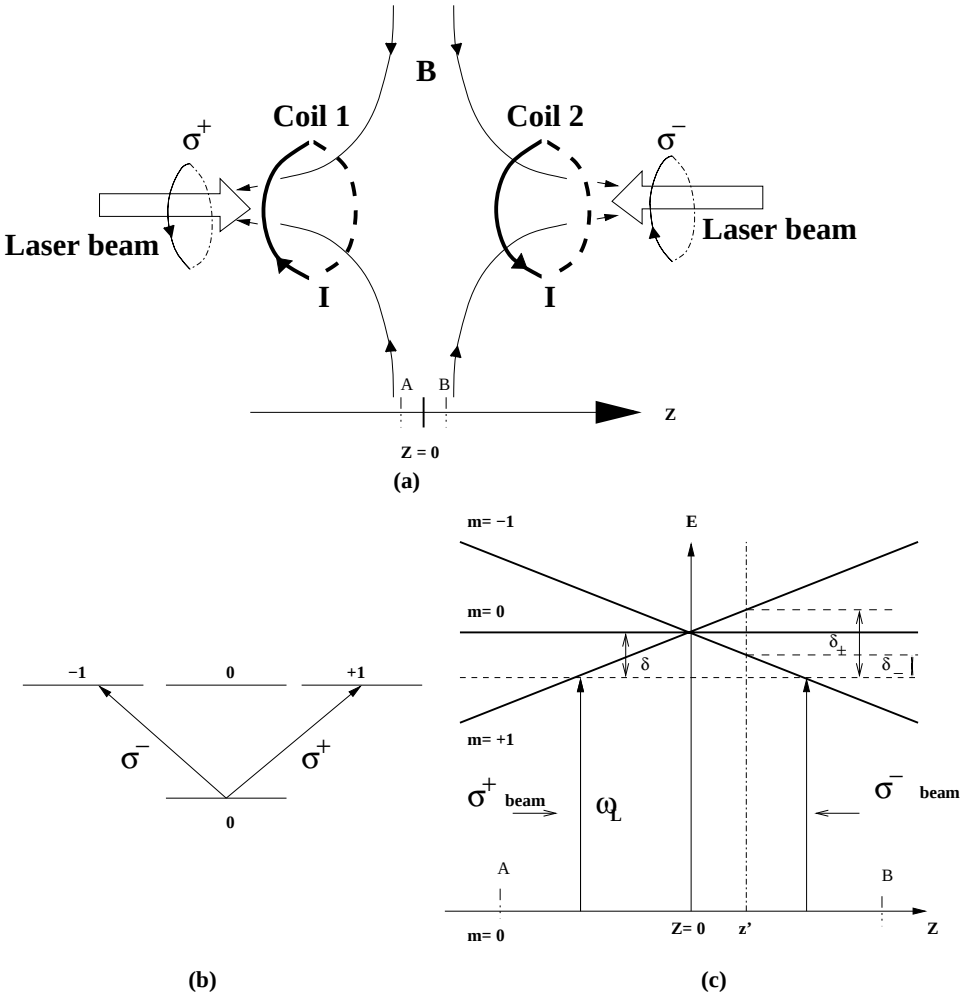

Figure 1.8: (a) Schematic 1D-MOT. Two anti-parallel laser beams with opposite helicity propagate along the z-axis. The atoms are also exposed to an anisotropic magnetic field produced by two anti- Helmholtz magnetic field coils 1 and 2. (b) For an atom with a transition from $J = 0 \rightarrow J = 1$ under action of circularly polarized light. (c) Energy Levels - The anisotropic field results in a spatially varying energy shift for different magnetic sub-levels. At the center B = 0 and the levels are degenerate.

onance in both cases, compression and cooling of the atoms is obtained simultaneously in a MOT. An explicit expression of this force can be given for low laser intensity: The total force on an atom in the low intensity regime is the sum of the forces exerted by the individual beams

$$F_{MOT} = F_{\sigma^+} + F_{\sigma^-} = \hbar k \frac{\Gamma}{2}\left[\frac{\frac{\Omega^2}{2}}{(\delta + \mathbf{k}\cdot\mathbf{v} - (\mu\beta z/\hbar))^2 + \frac{\Gamma^2}{4}} - \frac{\frac{\Omega^2}{2}}{(\delta - \mathbf{k}\cdot\mathbf{v} + (\mu\beta z/\hbar))^2 + \frac{\Gamma^2}{4}}\right] \quad (1.28)$$

where βz is the frequency shift due to the magnetic field. For small velocity and a Zeeman shift comparable to Γ

$$F(v) = \hbar k \frac{\Gamma}{2}\frac{\Omega^2\delta}{(\delta^2 + \frac{\Gamma^2}{4})}(kv + (\mu\beta z/\hbar)) \quad (1.29)$$

This expression is identical to the expression in optical molasses except that kv has been replaced by $(kv + (\mu\beta z/\hbar))$. The motion of an atom under the influence of this force is that of a damped harmonic oscillator

$$\ddot{z} + \gamma\dot{z} + \omega_{trap}^2 z = 0 \quad (1.30)$$

The Magneto-Optic Trap cools the atoms as in an optical molasses and in addition also confines the atoms to the trapping region around z = 0. Figure 1.9 shows numerical simulation of damping of the velocity of an atom and its oscillation about the center. For simulation, we have chosen the size of the MOT 4mm, magnetic field gradient 10G/cm, detuning of the laser beam -4Γ, intensity equal to the saturation intensity.

The MOT scheme can easily be extended to 3D by using six, instead of two, laser beams. This is shown schematically in figure 1.10. Moreover, even though very few atomic species have transitions as simple as $\mathbf{J}_g = 0 \rightarrow \mathbf{J}_e = 1$, the scheme works for any $\mathbf{J}_g \rightarrow \mathbf{J}_e = \mathbf{J}_g + 1$ transition. Atoms that scatter mainly from the σ^+ laser beam will be optically pumped toward the $m_g = +J_g$ substate, which forms a closed system with the $m_e = +J_e$ substate. Same happens with σ^- which pumps the atoms to m_g = -J_g which forms a closed system with $m_e = -J_e$ substate.

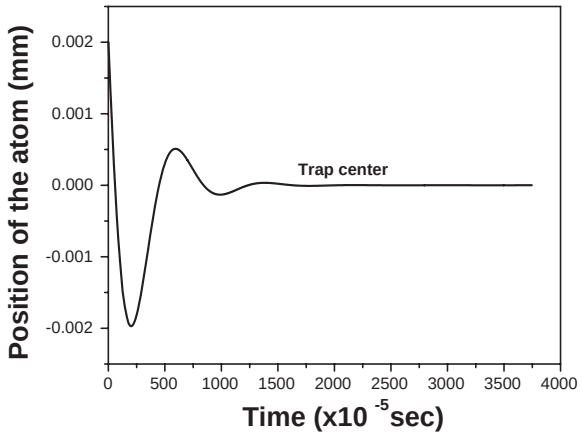

Figure 1.9: Plot to show how the velocity of an atom damps in a MOT and the atom oscillates about the center of the MOT

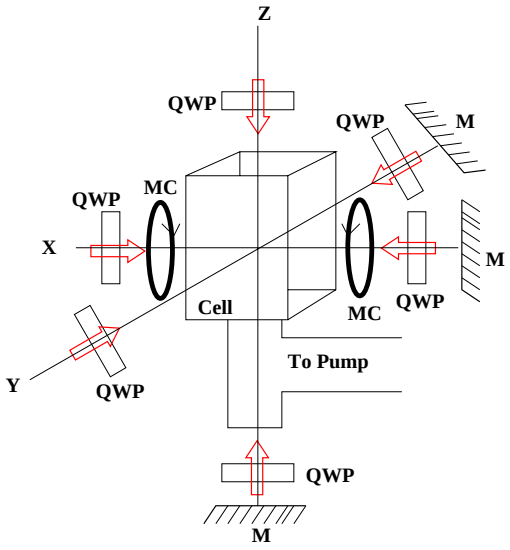

Figure 1.10: A schematic arrangement of a MOT. QWP: quarter wave plate; M: Mirror, MC: Magnetic coils.

The restoring force in the MOT can be determined by one of two methods. One can push the cloud of atoms a known distance away from the center. When the push beam is shut off the atoms come back to the center and one can measure the time taken by the atoms to return to the center. A second method is to use a small AC magnetic field to cause the atom cloud to vibrate in the trap. One can measure the phase of the vibration relative to the phase of the magnetic field by using a photo detector and a lock in amplifier. As the frequency of the oscillating field is changed, the phase will increase. The variation of phase with frequency can be fitted to a damped harmonic oscillator model to get the restoring force constant as well as the damping coefficient. This method was first used by Khons et al [17]. We have used this method to measure the force constant and hence the temperature of the cloud. This is described in chapter IV.

It is possible with a MOT to directly trap sufficient number of low velocity atoms from Maxwellian tail of a vapour beam of Rb or Cs without using a slowing down beam as shown by Monroe et al [18].

The maximum density in a MOT is of the order of 10^{11}/cc and the maximum number of atoms captured in MOT is of the order of 10^{8} atoms.

To summarise, in this chapter, we have discussed the salient features of the MOT which will be relevant to the work reported in this thesis. The above review is not intendened to be exhaustive. It is intended to present facts salient to the studies reported in this thesis. For a more complete discusion of laser cooling and trapping one may refer to the book by Metcalf et al [2] and the article by Foot [20].

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