Chapter 2

Pulsar Statistics

2.1 Introduction

The birth and the life history of radio pulsars have been the topic of wide interest ever since the discovery of pulsars. To cite a couple of pioneering efforts, Gunn & Ostriker (1970) discussed the birth rate of pulsars, their initial velocities, the threshold mass for neutron star formation, etc.. The distribution of pulsars as a function of the rotation period and its time derivative was discussed by Lyne, Ritchings & Smith (1975). A few important issues like the magnetic field decay time scales etc. were also discussed by them. The distribution of pulsars in the Galaxy, and the effect of observational selection effects were discussed by Taylor & Manchester (1977).

The evolution of pulsars as a function of period and its time derivative was discussed in papers by Phinney & Blandford (1981), and Vivekanand & Narayan (1981). The concept of pulsar current was developed in these two papers. In their study Vivekanand & Narayan (1981) arrived at one interesting conclusion that a significant fraction of pulsars may be born rotating slowly. They named this phenomenon as ***injection.*** A similar conclusion was arrived at by Srinivasan, Bhattacharya & Dwarakanath (1984) in their study of flat spectrum filled centre supernova remnants such as the Crab nebula. Two more recent papers by Narayan (1987), and Narayan & Ostriker (1990) dealt with issues like the birth rate of pulsars, magnetic field decay time scale *etc.* Also, these two papers describe an unbiased method of compensating for all the observational selection effects. which enables one to study the properties of the true population of pulsars more reliably.

This chapter describes yet another statistical analysis of pulsar population. The important difference between the earlier works and this work is that a better distance model (Taylor & Cordes 1993), and the modified radio luminosity model of Narayan & Ostriker (1990) have been used. Apart from estimating the birth rate of pulsars (with the knowledge of more reliable distance model and luminosity model), we wish to revisit the question of injection of pulsars and suggest **that** the injected pulsars may be the *recycled pulsars from massive and intermediate mass binaries.* This suggestion provides a way to estimate the fraction of recycled (solitary) pulsars in the general population of pulsars.

Section 2.4 described the estimation of birth rate of pulsars. The analysis which leads to the conclusion that the injected pulsars may be the recycled pulsars from massive and intermediate mass range binary systems is described in section 2.5. The last section summarises the conclusions of this chapter and includes a discussion of the results.

2.2 The Pulsar Current

The pulsar current in a period bin of width ΔP around a period **P** can be formally defined as (Phinney & Blandford 1981; Vivekanand & Narayan 1981),

$$J(P) = \frac{1}{\Delta P}\left(\sum_{i=1}^{N_{\rm psr}} \dot{P}_i\right) \qquad (2.1)$$

where $N_{\rm psr}$ is the number of known pulsars in the bin, and $\dot{P}_i$ is the time derivative of the rotation period of the i$^{\rm th}$ pulsar. However, it turns out that the distribution of rotation periods of the known pulsars are severely biased due to various observational selection effects. Therefore the above expression must be suitably modified to account for all the observational selection effects. The procedure followed to model the selection effects is given in the following section.

2.3 Selection Effects

The observed distribution of pulsars in the Galaxy differs systematically from the true distribution due to various observational selection effects. These selection effects can be summarised as follows (for original references, see Narayan 1987; Narayan & Ostriker 1990).

Survey parameters: Whether or not **a** given pulsar survey would have detected a pulsar of period P in a given region in the sky depends on (i) whether the survey has looked at that region, (ii) whether the survey is sensitive enough to detect it, and (iii) whether the sampling rate of the receiver satisfies the Nyquist condition to detect the pulse frequency.

Radio luminosity: Since pulsars which are far away from us are fainter the probability of detecting those pulsars is less than of detecting the pulsars which are nearer. Radio luminosity of pulsars has been modelled as a function of the observable parameters P and P. This will be described in detail later in this section.

Interstellar scattering: The scattering in the interstellar medium is caused by the free electron density fluctuations. This broadens the pulse profile resulting in the reduction of the peak flux.

Dispersion: The interstellar dispersion of radio waves causes the pulsar signal at different frequencies to arrive at different times. Higher frequency signal arrives earlier than the lower frequency signals. Since the frequency channels in the receiver have 'non-zero' width, the dispersion within the frequency range of the channels causes broadening of pulse profiles, which again results in the reduction of peak flux.

Beaming: Since the radio emission from pulsars are not radiated isotropically, whether not not one receives pulsar signals depends on whether the line of sight intersects the emission beam. The fractional solid angle of the sky in which the emission is beamed is modeled by many authors, and in the standard model this fraction is about 0.2.

For the work described in this chapter, the sample set of pulsars is restricted only to those which in principle could have been detected by one of the following eight major pulsar surveys: (1) Jodrell Bank survey (Davies *et al.* 1972), (2) U.Mass–Arecibo survey (Hulse & Taylor 1974), (3) Second Molonglo survey (Manchester *etal.* (1978), (4) U.Mass–NRAO survey (Damashek, Taylor & Hulse 1978), (5) Princeton–NRAO Phase I survey (Dewey *et al.* 1985), (6) Princeton–NRAO Phase II survey (Stokes *et al.* 1985), (7) Princeton–Arecibo survey (Segestein *et al.* 1986), and (8) Jodrell Bank–1400 MHz survey (Clifton & Lyne 1986). There are about 325 pulsars which satisfy this condition. The Globular Cluster pulsars, the Extragalactic pulsars, and the millisecond pulsars are not included in this analysis.

The *Detection Probability* is the ratio of the observed number of pulsars to the *true* number. For a given pulse period P and a luminosity L at a height z from the Galactic plane, this can be defined as,

$$Det.Prob.(P, L, z) = \frac{\int\int_{\rm obs} \rho_R(R)\, R\, {\rm d}R\, {\rm d}\phi}{\int\int \rho_R(R)\, R\, {\rm d}R\, {\rm d}\phi} \tag{2.2}$$

Here, R and ϕ are the galactocentric radius and the azimuth angle respectively. The function p describes the distribution of pulsars with respect to the Galactocentric radius. The integral in the denominator is over a volume of a slab of the Galaxy at a height z from the Galactic plane, while that in the numerator is only over a subset of the above volume of the Galaxy where a pulsar of period P and luminosity L can be detected by atleast one of the above given eight surveys. The *scale factor* is defined to be the reciprocal of the detection probability.

The detection probability for a given period (P), luminosity (L) and the height from the plane (z) was computed by generating a large number of objects in the Galaxy with an assumed distribution in R (a gaussian with a = 4.5 kpc), and determining the fraction of pulsars detected by any one of the eight surveys mentioned earlier.

The broadening of the pulse profile due to the interstellar scattering was computed by the free electron density distribution model[1] by Taylor & Cordes (1993). The pulse broadening due to the interstellar dispersion was computed with the knowledge of the column density of free electrons[2] and the specifications of the receivers used in the individual surveys. With the information of the area coverage of the surveys, and their respective sensitivities, the detection probability as

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[1]This is a model for the distribution of free electrons in the Galaxy. It has four different components namely: (1)the central bulge. (2) the disk, (3) spiral arms, and (4) the Gum nebula.

[2]The column density of free electrons (called *dispersion measure)* is defined *as* $\int_0^d n_e$ dl, where n_e is the number density of free electrons, d is the distance to the pulsar, and dl is the distance element along the line of sight.

a function of (P,L, z) is computed. This is the fraction of pulsars which are in principle *detectable* by atleast one of these eight surveys.

Now, with the knowledge of detection probability as a function of (P, L, z) one can evaluate detection probability as a function of $(P, \dot{P}, z)$, with the help of a radio luminosity model of pulsars (It should be pointed out that since P and $\dot{P}$ are the measured quantities, one needs the det. prob. as a function of these parameters, rather than P and L). The model by Prózyński & Przybycień (1984) was used for this purpose, which describes the radio luminosity of pulsars as a function of the rotation period (P) and its time derivative $(\dot{P})$ (see figure 2.1),

$$\log L_m \;=\; \frac{1}{3}\log\left(\frac{\dot{P}_{-15}}{P^3}\right) + 1.635 \qquad (2.3)$$

Here, $\dot{P}_{-15}$ is the time derivative of the period in units of 10^{-15} s/s. However, though this luminosity model gives an average luminosity for pulsars as a function of P and $\dot{P}$, in reality pulsars seem to have a significant spread around the model luminosity. This is not surprising since this is only an empirical fit to the data. Therefore it is essential to account for this spread explicitly while converting from the variables (P, L, z) to $(P, \dot{P}, z)$. There are two methods by which this can be done. The first method can be summarised by the following equation.

$$S(P,\dot{P},z) \;=\; \int S(P,L,z)\;\rho(\chi)\;\mathrm{d}\log L \qquad (2.4)$$

where the *scale factor* $S(P, L, z)$ which is the reciprocal of the detection probability is averaged with the luminosity spread function $\rho(\chi)$, Here, $\chi = [\log L - \log\langle L_m(P,\dot{P})\rangle]$, where $\langle L_m \rangle$ is the model luminosity. The second way is to average the detection probability itself with the luminosity spread function since this is the more fundamental quantity. According to this prescription.

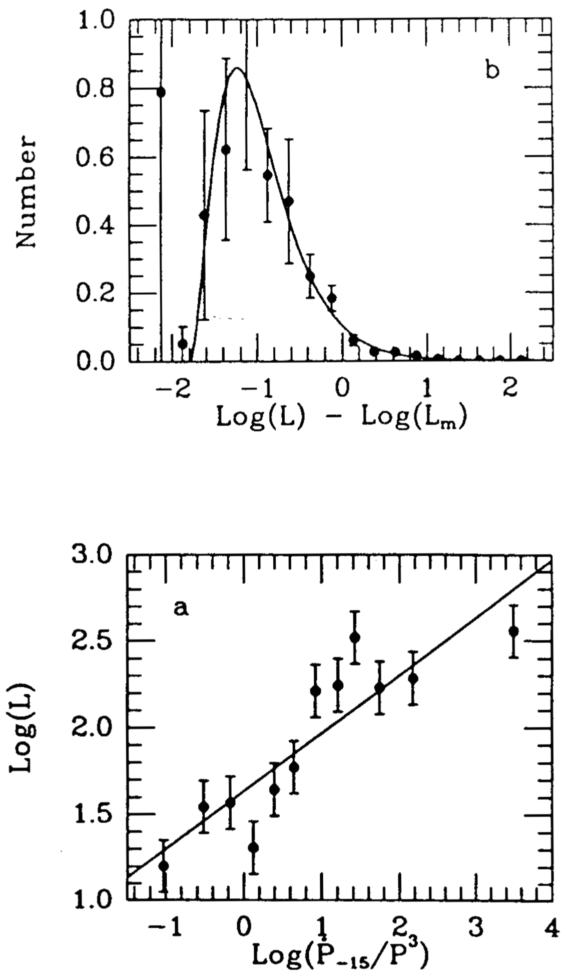

Figure 2.1: (from Narayan & Ostriker 1990) The first panel gives the radio luminosity model of pulsars. Filled circles with error bars are the mean luminosities of observed pulsars binned as a function of $\log(\dot{P}_{-15}/P)$. The solid line corresponds to the model luminosity given by Prózyński & Pryzybycien (1981) (see equation **2.3)**. The second panel gives the luminosity spread function. The filled circles are from the observations after correcting for the observational selection effects. and the solid line is the model fit.

$$< det.prob. > = \int det.prob.\ \rho(\chi)\ \mathrm{d}\log \mathrm{L} \tag{2.5}$$

here, the function $\rho(\chi)$, which describes the spread in the observed radio luminosity with respect to the model luminosity is given by (Narayan & Ostriker 1990; Bhattacharya et al. 1992),

$$\rho(\chi) \ = \ 0.5\ [\alpha_l(\chi + \beta_l)]^2\ \exp\left[-\alpha_l(\chi + \beta_l)\right] \quad \text{where } \alpha_l = 3, \text{ and } \beta_l = 2 \tag{2.6}$$

This function is plotted in figure 2.1. Noting that the luminosity spread function is a probability distribution function, it is clear that the procedure given in equation 2.5 is more appropriate. Hence, the second method is adopted for the whole of this analysis. Here, it may be noted that the scale factor $S(P, \dot{P}, z)$ is the reciprocal of the RHS of equation 2.5.

Modified current distribution

With the knowledge of the scale factors and the beaming fraction, the equation for pulsar current given in equation 2.1 may be modified as,

$$J(P) \ = \ \frac{1}{\Delta P} \sum_{i=1}^{N_{\rm psr}} S_i \dot{P}_i \frac{1}{f} \tag{2.7}$$

here, S_i is the scale factor corresponding to the i^{th} pulsar, f is the beaming fraction[3]. If the current in a particular bin has reached its maximum value, *i.e.*, if the initial periods of pulsars are less than the period corresponding to the bin, and if the death[4] of pulsars has not yet set in, then the maximum value of the current represents the birth rate of pulsars.

[3]Since pulsars emit radio emission in a cone, the emission is beamed to only a fraction of 4π sterradians of the sky. The beaming fraction is defined as the fractional solid angle in which the emission is beamed. In the standard model, the value of this fraction is about 0.2

[4]Pulsars seem to "die" after functioning for a few tens of Myr. *i.e.*, they cease to function as pulsars. In the pulsar emission mechanism model of Sturrock (1971) and Ruderman &

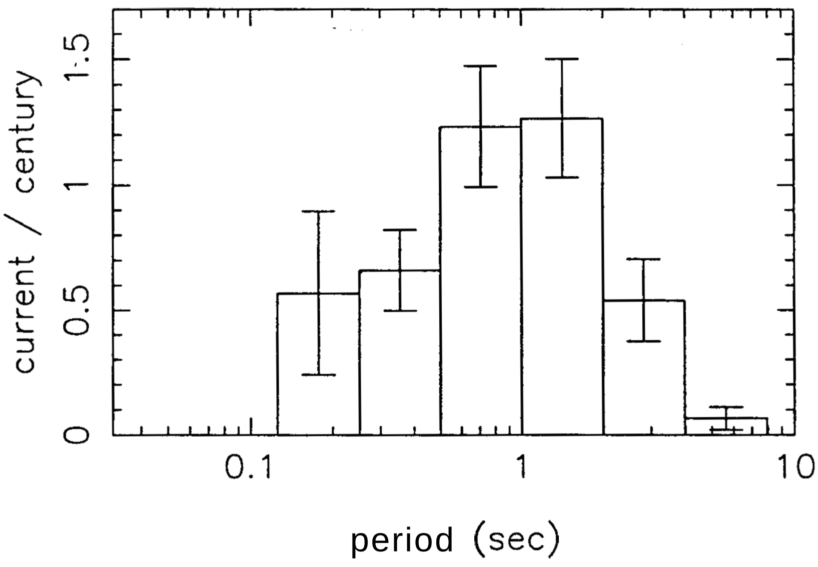

Figure 2.2: The current distribution as a function of period. As may be seen, the current reaches its maximum value around a period of about 0.5 sec. It stays constant upto a period of about 2 sec. The maximum value of the current corresponds to a pulsar birth rate of about one pulsar in 75 ± 15 years.

2.4 The Birth rate of Pulsars

As explained in section 2.3 the steady value of the pulsar current is a measure of the birth rate. Figure 2.2 shows the pulsar current as a function of the rotation period. It can be seen from the figure that the current continues to rise till a period of about $\sim$ 0.6 sec. A straight-forward interpretation of this is that not all pulsars are born spinning as rapidly as, say, the Crab pulsar. The current is roughly constant in the period range of 0.6–2 sec. Beyond a period of $\sim$ 2 sec. the current decreases presumably because the deaths of pulsars begin to become important. The maximum current yields a birth rate of about 75 ± 15 years (the error indicated is only the statistical error). This should be compared with one in about 100 years derived by Narayan & Ostriker (1990), and one in about 125–250 years for a luminosity limited set of samples by Lorimer et al. (1993).

	Old Dist. model (per century)	New Dist model (per century)
Old Lumin. model	0.32	0.15
New Lumin. model	2.8	1.4

Table 2.1: Galactic birth rate of pulsars obtained by old and new distance model and luminosity model. Ref. (1) Narayan (1987), (2) Narayan & Ostriker (1990), (3) Lyne et al. (1985), and (4) Taylor & Cordes (1993).

To understand the sensitivity of the derived birth rate to the assumed distance model (Taylor & Cordes 1993) as well as the luminosity model (Prózyński & Przybycień 1984; Narayan & Ostriker 1990), the current analysis was repeated

Sutherland (1975) when the voltage generated by the pulsar (which is proportional to B/P^2) drops below a critical value ($B/P^2 \approx 2 \times 10^{11}$), copious pair production will cease and the pulsar will stop functioning. This critical line ($B/P^2 = 2 \times 10^{11}$), which is called death *line,* is drawn in figure **2.3.**

using the luminosity model of Narayan (1987), and the distance model[5] of Lyne *et al.* (198.5). The corresponding birth rates are also given in table 2.1. As one can see, the *old luminosity model severely underestimates the birth rate.* The reason for this is that the old luminosity model does not take into account of selection effects while modeling the spread around the model luminosity. Therefore the model is biased towards brighter pulsars. The revision in the birth rate due to the change in the distance model is roughly a factor of two. However, it is not clear how complete is the new distance model by Taylor & Cordes (1993).

While the above given birth rate estimate is in reasonable agreement with that of Narayan & Ostriker (1990), the estimate of Lorimer *et al.* (1993) is somewhat on the lower side. Lorimer *et al.* considered only pulsars with radio luminosities greater than 10 mJy kpc^2. The recent discovery of PSR J0108−1431 (Tauris *et al.* 1994) indicates that active pulsars with radio luminosities as low as 0.06 mJy kpc^2 may not be uncommon. Tauris *et al.* have argued that the number of pulsars in the Galaxy similar to PSR J0108−1431 could be at least 5 x 10^5. The inclusion of such a population will increase the birth rate substantially. However, the calculation of birth rate involves some assumptions like the field decay time scale *etc.* This is because, for calculating the birth rate, one needs the knowledge of the ages of pulsars. If the magnetic field decay time scale is considerably shorter than the average life time of pulsars, it turns out that the characteristic age (which can be calculated by $P/2\dot{P}$) is not a good indicator of the actual age (the actual age will be smaller than the characteristic age). **A** short field decay time scale of about

[5]The distance model of Lyne *et al.* (1985) has three components. The first component is a layer whose scale height is much more than that of pulsars themselves, the second component is a thin layer with scale height of 75 pc, and the third component is the Gum nebula. The main difference between this model and the recent model by Taylor & Cordes (1993) is that the latter has modeled the spiral arms of the Galaxy as a separate component. Also, on the average, the distances predicted by the latter model is more than the value predicted by the former.

5 Myr as assumed by Tauris *et* al. will revise the birth rate upwards to one in about 40 years. However, since the recent statistical studies (Bhattacharya *et* al. 1992) seem to suggest longer field decay time scales, if one assumes a reasonably longer decay time scale of about 25 Myr will give a birth rate of about one in 120 years. Therefore one can conclude that there is no serious discrepancy between the birth rate estimate given above and that of Lorimer *et* al. (1993).

The pulsar birth rate should ideally match that of the supernovae that result in the formation of neutron stars. The core collapse supernova rate (Type Ib+II) inferred from the observations of external galaxies (van den Bergh 1991) of morphology similar to ours is about 1.6 per century per $10^{10} L_{\odot}(B)$ (about one in 60 years). Using the Initial Mass Function of Scalo (1986) and the population-I star model of Ratnatunga & van den Bergh (1989), van den Bergh (1991) estimates the Galactic core-collapse supernova rate to be one in about 100 years or so. Given the uncertainties in the determination of the birth rate of pulsars as well as the supernova rate one can not consider them to be very discrepant.

If one tries to compare the pulsar birth rate estimated above with some of the earlier estimates, one can find that this number has fluctuated significantly over past two decades or so. Gunn & Ostriker (1970) estimated the birth rate to be one in about 30 years. However, a few years later Vivekanand & Narayan (1981) estimated this value to be one in about 20 years. The value estimated by Phinney & Blandford (1981) too was not very different from this value. Lyne, Manchester & Taylor (1985), through their statistical analysis, estimated the birth rate to be one in about 30 – 120 years. But due to various refinements in the distance model *etc.*, the value estimated by Narayan (1987) was one in about 60 years. Then, in another statistical analysis, Narayan & Ostriker (1990) have estimated this value to be one in about 100 years or so. Keeping all the uncertainties in mind. one

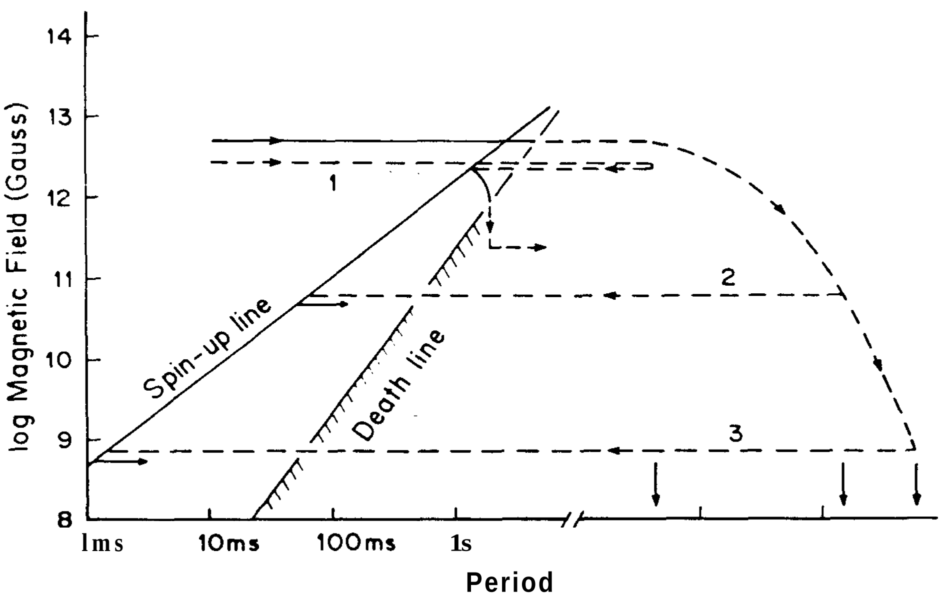

Figure **2.3:** (from Srinivasan et al. 1990) Three possible evolutionary scenarios of recycled pulsars are shown here. The tracks **1**, **2** and **3** correspond to the life history of the first-born neutron star in massive binaries, intermediate mass range binaries, and low mass binaries, respectively. The first-born neutron star in the low mass binary systems are supposed to undergo many orders of field decay due to the long spin-down phase due to the stellar wind from the main sequence companion, whereas, in the case of the first-born neutron stars in massive binaries, since the spin-down phase is short-lived, there isn't any significant field decay. They get spun-up into the main island of pulsars. Track 2 describes the intermediate situation.

may conclude that the birth rate of pulsars may be one in about 50–100 years.

2.5 Pulsars from Binary Systems

Although the number of known binary pulsars is only a couple of dozen, there is no reason to conclude that the vast majority of the solitary pulsars may not have come from binary systems. After all, one expects the majority of the binaries to disrupt during supernova explosions. The question of estimating the fraction of pulsars that come from binary systems was first raised by Radhakrishnan & Srinivasan (1981). If the magnetic field of the first born pulsar had decayed significantly in the time interval between its birth and the onset of mass transfer from the companion, then it will be spun-up to relatively short periods and will stand out from the general population of pulsars, like PSR 1913+16 (Appendix A gives a brief description of the scenario of recycling in binary systems). Based on this criterion, PSR 1541–52 and PSR 1804–08 were tentatively identified by Radhakrishnan & Srinivasan (1981) as recycled pulsars. However, if the magnetic field of the first-born pulsar had not decayed significantly, then the pulsar will be spun-up, and be deposited in the place where the majority of pulsars are found (*i.e.,* $\log B = 11.5 - 13$). It now appears that the magnetic fields of solitary neutron stars do not decay significantly during their lifetime as pulsars. But there are strong reasons to believe that the magnetic fields of neutron stars born and processed in binary systems do decay (Srinivasan et *al.* 1990; Bhattacharya et al. 1992). In the model due to Srinivasan et al. (1990) the magnetic field decay is related to the expulsion of magnetic flux from the interior of the star, as the neutron star is spun-down during the main sequence phase of the companion. If this scenario is correct, then there are three possibilities as shown in Figure **2.3.** In the case of Low Mass X-ray Binaries (LMXBs), which are presumably the progenitors

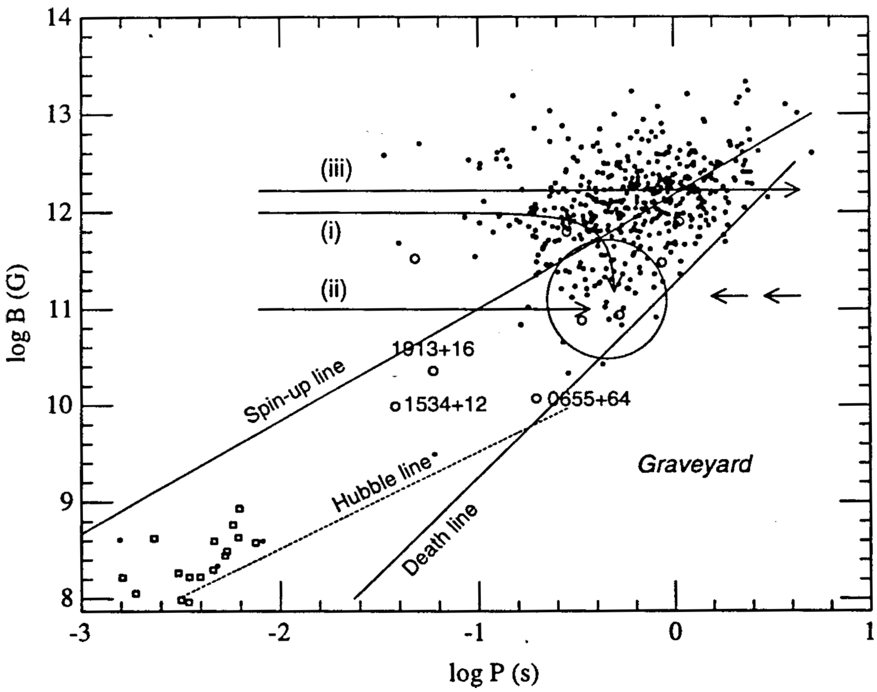

Figure **2.4:** The distribution of observed pulsars. Track marked (i) indicates the possibility that pulsars in the encircled region may have started with high field, and ended up there due to rapid field decay; track marked (ii) indicates the second possibility that they are born with low fields. Track marked (iii) indicates the recycling hypothesis. Pulsars, which evolve in intermediate mass binaries get recycled, as indicated by the two arrow marks.

of millisecond pulsars, the neutron stars are possibly spun-down sufficiently for the field to decay to very low values $\sim 10^8$ G (Jahan Miri & Bhattacharya 1994). In the case of massive and tight binaries the companion may evolve so fast that there may not have been any time for the flux to be expelled from the interior, let alone decay in the crust. This is scenario 1 in figure 2.3. In the case of the intermediate mass range binaries, the spin-down and the consequent flux expulsion may be less pronounced as shown in the alternative(2).

2.5.1 Evidence for injection in the low field range $10^{10} - 10^{11.5}$G

Let us first concentrate on the pulsars with fields less than about $10^{11.5}$ G (encircled in figure 2.4). There are two possible ways by which these pulsars could have evolved to their present positions in the $\log B - \log P$ diagram. Let us first consider the possibility that these pulsars could have evolved to their present positions from the left. This admits two alternative scenarios: (i) Their fields are relatively low because of rapid field decay, or (ii) they were born with low fields. The first alternative, that they are old aged Crab or Vela pulsars is inconsistent with the present estimates of the field decay time scales according to which $\tau_d \gtrsim 40$Myr (Bhattacharya et *al.* 1992; Kulkarni 1986). Infact, it is this scenario (i) that led one to erroneously conclude earlier that magnetic fields of neutron stars decay rapidly (Radhakrishnan & Srinivasan 1981; Radhakrishnan 1982). However, many of the recent works have concluded that the magnetic field decay time scale may be about 100 Myr or so. The second alternative viz., that they were born with short period and low magnetic field can not be ruled out so easily. Though one does not see pulsars in the period range of a few hundred milliseconds in this field range. it may very well be due to the small number statistics.

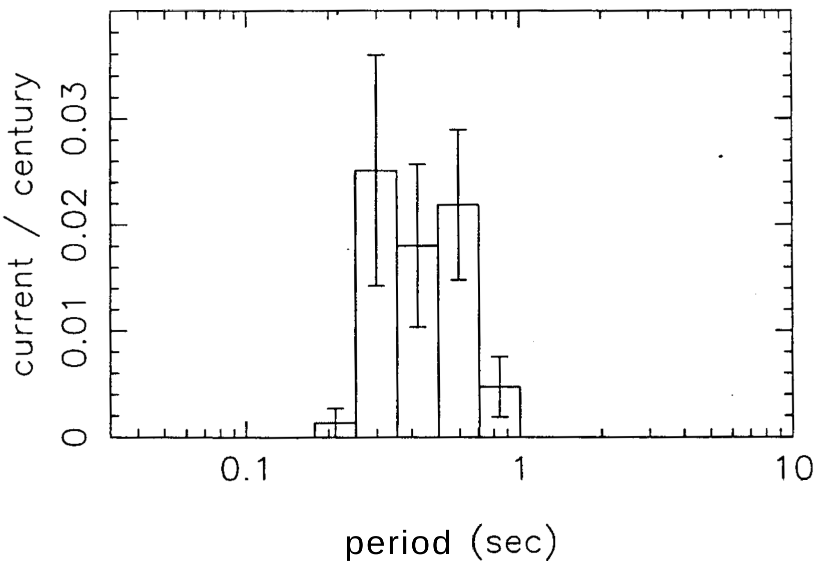

Figure **2.5:** The current as a function of period, in the field range of $logB = 10.5 - 11.5$. As may be seen, the birth rate of these pulsars turns out to be about one in 5000 years.

since one is dealing with very low birth rates, as may be seen in Figure 2.5. It suggests that one may be dealing with a birth rate as low as one in about 5000 years. Despite this possibility, we wish to suggest that these pulsars could have evolved to their present positions from the right of the diagram.

Figure 2.6 shows the distribution of true number of pulsars in the $P - B$ plane. The true number of pulsars in any given bin of width AP around a period P, and AB around a field B is given by,

$$N_B(P) = \frac{1}{f} \sum_{i=1}^{N_{\rm psr}} S(P_i, \dot{P}_i, z_i) \tag{2.8}$$

where f is the beaming factor and $S(P, \mathrm{P}, z)$ is the scale factor corresponding to the period **P**, period derivative **P**, and the height from the plane z, of the i$^{\rm th}$ pulsar. One sees that these low field pulsars appear to form a distinct island in the true number distribution shown in Figure 2.6, and there appears to be a valley between the two populations of pulsars. Many statistical tests were done to quantify the significance of this valley. For example, a valley was defined in the field range $11.5 < \log B(G) < 11.6$ and the period range from 0.1 sec. to the period at the death line. The number of known pulsars in the valley was counted. This number was taken as the reference. Then, (a) every pulsar was randomly assigned the luminosity of some other pulsar, (b) every pulsar was assigned a **P** of some other pulsar, and (c) the number of pulsars in the valley was counted. Steps (a) to (c) were repeated a large number of times ($\sim$ 10000). From the distribution of the number of pulsars in the valley, it is found that the significance of the reference number is 98.37%. As one can see, though the significance level is not extremely high, it is considerable.

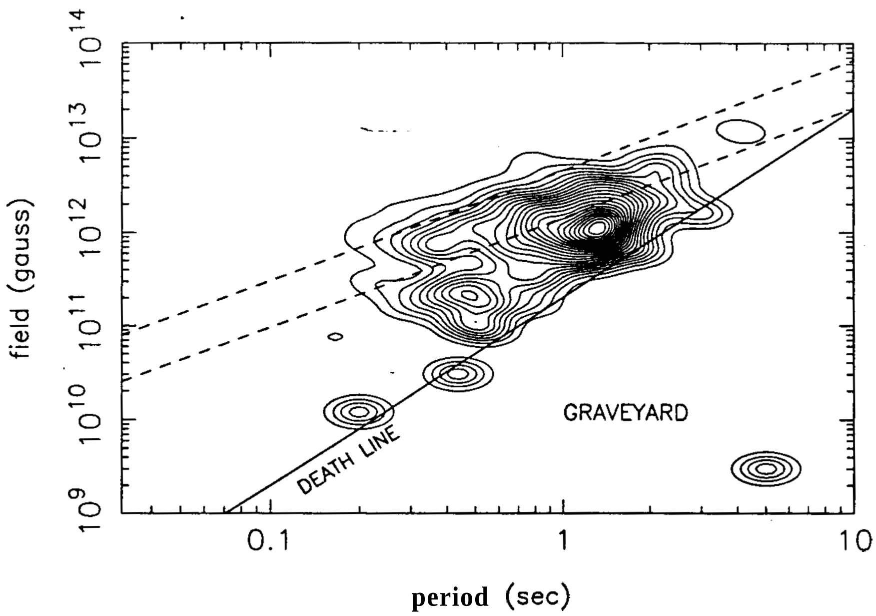

Figure 2.6: The *true* number distribution of pulsars in the B – P diagram, as defined by equation 2.8. The contours have been smoothed by a function indicated at the right hand side bottm of the figure. The pulsars in the field range of $\log B = 10.5 - 11.5$ seems to form a distinct island; there appears to be a valley between the distribution of these pulsars and the high field pulsars. As discussed in section 2.5.1 the statistical significance of this valley is 98.37%. The two 'dash' lines are equilibrium period lines. The lower one corresponds to the Eddington accretion rate, and the upper one, to ten times its value.

Their location close to and to the right of the spin-up line[6] (see figure 2.4) may suggest that in fact these pulsars are recycled pulsars. Additional support for this comes from the distribution of pulsars with respect to the Galactic plane. The birth rate of these relatively low field pulsars suggests that they constitute roughly **3%** of the total population.

2.5.2 High **Field Injection**

If at all there is ***injection*** of first-born pulsars (from massive binaries) in the high field population,then one expects to see a step in the current in the vicinity of the spin-up line. The integrated current of pulsars shown in Figure 2.2 shows such a step at a period of about 0.5 sec. If such an injection is not an artefact, and if it were due to recycled pulsars, then one would expect pulsars contributing to that step to have magnetic fields in a narrow range defined by the spin-up line. Figure 2.7 gives the current as a function of rotation period for three different ranges of magnetic fields. As may be seen, only in the narrow field range of $12 < \log B < 12.6$, there is some evidence of **a** step in the current. If one takes the step in the current seriously, then there appears to be a correlation between the magnetic fields and the periods of pulsars contributing to the step in the current, thus lending support to the suggestion that a reasonable number of recycled, but solitary pulsars are present in the main island of pulsars. It should be mentioned here that an injection of pulsars at a period of about 0.5 sec. was originally pointed out by Vivekanand & Narayan (1981). More recently Narayan & Ostriker (1990) noted that the injection occurs in a narrow field range. The new suggestion here is to relate the injection to the recycled pulsars.

[6] When a neutron star accretes matter frorn its binary companion. it gets spun-up (since some angular momentum is associated with the accreting matter. The final rotation period. which is called the ***equilibrium*** period, of the neutron star is determined by its surface magnetic field and the mass accretion rate.

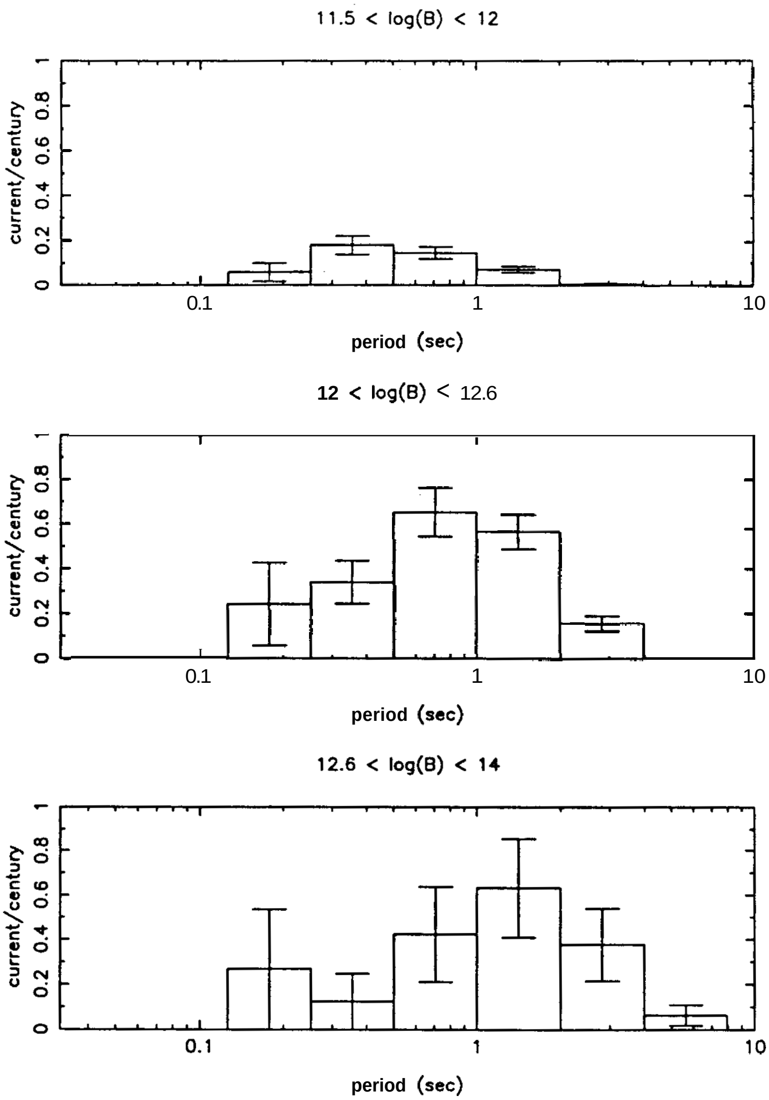

Figure 2.7: The distribution of current in three different field ranges. A step in the current can be seen at a period of about 0.5 sec., in the central panel. which corresponds to a field range of $\log B = 12 - 12.6$.

The correlation between the rotation period and the magnetic field can be seen better in Figure 2.8 where the current is plotted as a contour diagram in the $B - P$ plane. This current is calculated using equation 2.7 in various magnetic field bins. Concentrating for a moment in the field range of $10^{12} < B < 3 \times 10^{12}$ Gauss, one can see that the current builds up continuously till a period of about 0.5 sec., at which there is a step or a cliff (as may be readily seen, the contour plot reveals many 'hills'. These are merely individual high P pulsars which appear as 'little hills' due to the fact that the current distribution has been smoothed with the function shown in the right hand bottom corner of the plot). Contrary to this, the step in current referred to above, is a statistically significant feature since a fairly large number of pulsars contribute to it. Also shown in the figure are two equilibrium period lines corresponding to the Eddington accretion rate, and ten times its value. Therefore if the injection is interpreted as due to the recycled pulsars, since the rise in current occurs pretty close to the equilibrium period line corresponding to ten times the Eddington-rate, it would imply that the neutron stars experience accretion at super-Eddington rate during their spin-up phase. This may be quite likely to happen in massive binary systems.

2.5.3 Birth Places of Injected Pulsars

Further support for the above conjecture comes from the distribution of pulsars with respect to the Galactic plane. Pulsar current as a function of characteristic age ($\tau_{ch} = P/2\dot{P}$) and the distance z from the Galactic plane can be defined as,

$$J_z(\tau(P,\dot{P})) = \frac{1}{f}\frac{\sum_{i=1}^{N_{\rm psr}} S(P,\dot{P},z)}{\Delta\tau_{ch}} \qquad (2.9)$$

Figure 2.9 gives the current distribution in the $z - \tau_{ch}$ plane for the field range $12 < \log B < 12.6$, and a range which excludes this (*i.e.*, $9 \leq \log B \leq 12$ and

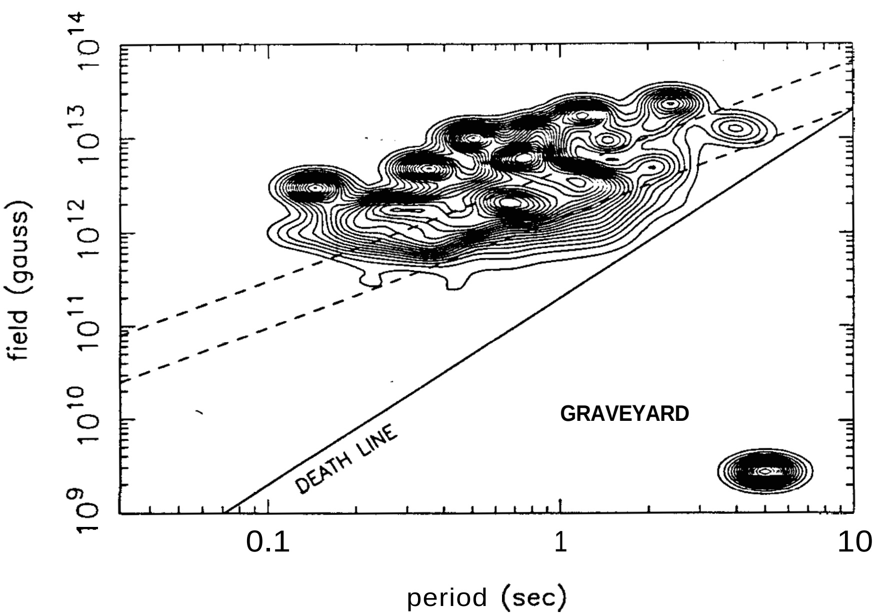

Figure 2.8: Current distribution as a function of period and magnetic field. The distribution has been smoothed by the function given in the right hand bottom corner of the diagram. Most of the 'hills' in the diagram correspond to the high **P** pulsars. But there is a distinct cliff in the field range of $10^{12} < B < 10^{12.6}$ and **a** period around **0.5** sec. It is this step in current close to the upper spin-up line ($M = 10\ \dot{M}_{Edd}$) that is associated with the injection of recycled pulsars from massive binaries. The two 'dash' lines are equilibrium period lines. The lower one corresponds to the Eddington accretion rate, and the upper one, to ten times its value.

$12.6 \leq \log B \leq 14$). As one can see, the current corresponding to the field range of $\log B = 12 - 12.6$ seem to be distributed upto considerable heights from the Galactic plane. Strictly going by the definition of current, this would imply that these pulsars are born at various heights from the plane.

What could be the progenitors of these pulsars? The hypothesis that these injected pulsars are the recycled pulsars from binary systems offers a natural explanation to this. It is conceivable that a certain fraction of binaries acquire substantial centre of mass velocities during their first explosion, and a fraction of them migrate away from the Galactic plane. If the secondary star of these systems is massive enough, it will explode as a supernova, producing a neutron star. The first-born will be a recycled pulsar, and the second one will be an ordinary pulsar. This would offer a natural explanation for why some short characteristic age pulsars are observed moving towards the Galactic plane (Harrison et al. 1993).

2.6 Discussion

If the hypothesis that the injected pulsars are the recycled pulsars from binary systems is correct, then a plausible evolutionary scenario is as follows: Massive binaries are presumably born close to the plane. After the primary star evolves and explodes, if the binary survives, the centre of mass of the binary system will acquire some space velocity since the explosion is not symmetric with respect to the centre of mass of the binary system. A fraction of these binaries can migrate to substantial heights from the Galactic plane. The subsequent evolution of these binaries depends upon several factors, especially the orbital parameters. If the orbital period is greater than about a year, then after the spiral-in phase. such systems will become very tight binaries consisting of the first-born neutron star and the helium core of the companion. If this core is massive enough. it will

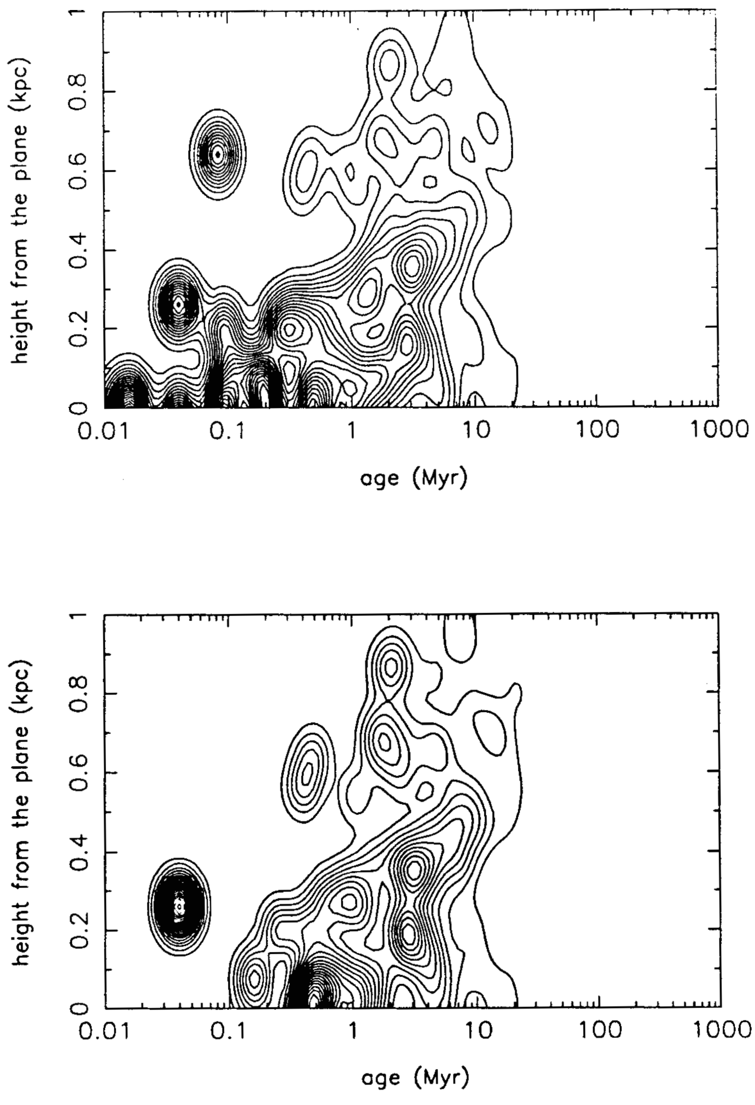

Figure 2.9: Pulsar current as a function of characteristic age and the height from the galactic plane. The first panel excludes pulsars in the field range of $\log B = 12 - 12.6$, and the second panel gives the current distribution for only those pulsars in the field range of $\log B = 12 - 12.6$. As can be seen, the distribution of current in the second panel goes all the way upto one kpc.

explode as a supernova and produce a neutron star. Most probably the binary will disrupt in this process, releasing two run-away pulsars: the first-born recycled pulsar, and the second-born pulsar which will be indistinguishable from the one born from a solitary progenitor. This kind of binaries can naturally account for some of the short characteristic age pulsars which are at considerable distance from the plane and which are moving towards the Galactic plane (Harrison et al. 1993). On the other hand, if the binary is sufficiently tight to begin with (orbital period shorter than about a year), the common envelope formed during the spiral-in phase may not be expelled, and the first-born neutron star may actually spiral into the core of the companion star. The outcome of these systems will be a single recycled pulsar (for a detailed review, see Bhattacharya & van den Heuvel 1991). Thus, the binary hypothesis offers logically consistent explanation for several things:

1. The injection of pulsars into the population of solitary pulsars.

2. The correlation between the rotation period of the injected pulsars and their magnetic field.

3. The large spread in their birth places with respect to the Galactic plane.

Regarding the actual fraction of injected pulsars into the main island of pulsars, one can estimate the following. In a sense, the magnitude of the step in the current at a period of —0.5 sec. itself is an indicator of this. But one must keep in mind that this could be an overestimate since part of this step could be due to the genuine spread in the initial periods of pulsars. Keeping this in mind, the upper limit to this fraction has been estimated to be about 10 – 15% (Deshpande, Ramachandran & Srinivasan 1995). *i.e.*, about 10 – 15% of the pulsars are processed in binary systems. Given the total birth rate of pulsars, this fraction would correspond to one recycled pulsar injected into the main island every thousand years. This would imply that the number of wide binaries with a neutron star and a B/Be star companion is about 10 – 1000 (corresponding to an average lifetime of such systems in the range $10^6 - 10^4$yr). The recently discovered pulsar PSR B1259−63 is one such system. The estimated distance to this system is about 4 kpc. A simple minded scaling suggests that there could be as many as 100 such systems in the Galaxy, which is certainly consistent with the expected numbers mentioned above.

Finally, the main conclusions of this chapter can be summarised as follows.

- The birth rate of pulsars in the Galaxy is one in 75 ± 15years.
- The pulsars with $B = 10^{10.5} - 10^{11.5}$G are interpreted as recycled pulsars from intermediate mass range binaries.
- There may be a substantial number of solitary recycled pulsars in the normal pulsar population (produced by solitary progenitors).

Appendix A

Recycling in binaries

The study of the origin of radio pulsars from binary systems started with a suggestion by Bisnovatyi-Kogan & Komberg (1974), that the pulsating X-Ray sources in X-Ray binaries may later in life become radio pulsars, after their companions have exploded as a supernova. This was quite a natural suggestion since some of the pulsating X-Ray sources known at that time, like SMC X-1 ($P = 0.71$ sec.) and Cen X-3 ($P = 4.84$ sec.) are getting spun-up rapidly on time scales of the order of a few thousands of years, and have spin periods which are typical of radio pulsars.

In 1974 the first binary radio pulsar, PSR B1913+16, was discovered. To the great surprise, it had a very abnormal combination of rotation period and magnetic field strength. It had a relatively short rotational period ($P \sim 59$ msec.) and a field strength which was lower by two orders of magnitude than the other known radio pulsars. This led Srinivasan & van den Heuvel (1982) to conclude that PSR B1913+16 could be a recycled (due to accretion from the companion) from a binary system.

By 1982, the first millisecond pulsar was discovered by Backer and his colleagues (Backer et al. 1982). This pulsar, PSR B1937+21, was spinning **twenty** times faster than the fastest know pulsar (Crab) at that time, and it had a magnetic field strength, which was four orders of magnitude less,than the other normal pulsars. Unlike PSR B1913+16, this pulsar was a solitary pulsar. Independently, Radhakrishnan & Srinivasan (1982) and Alpar *et* al. (1982) came up with a suggestion that even this pulsar originated in a binary system, and had got spun-up due to heavy accretion. Subsequent discoveries of millisecond pulsars, combined with the fact that most of these pulsars were in binary systems, proved these authors to be correct.

HMBPs and LMBPs

Binary pulsars fall into two broad classifications. Systems like B1913+16 fall into a class called High Mass Binary Pulsars (HMBPs), which presumably originate from High Mass X-Ray Binaries (HMXBs), and systems like B1953+29 fall into another class called Low Mass Binary Pulsars (LMBPs), which presumably originate from Low Mass X-Ray Binaries (LMXBs).

After the supernova explosion of the primary star, High Mass X-Ray Binaries have a neutron star and a massive companion. Before summarising the spin evolution of neutron stars in these systems let us introduce some definitions.

Light cylinder of a neutron star which rotates with angular velocity ω is defined as $r_{lc} = (c/\omega)$, where c is the velocity of light. This is the radius where an object will revolve around the neutron star with a tangential velocity equal to the velocity of light, if it is forced to corotate with the neutron star.

Alfvén radius is defined by the condition that the energy density of the magnetic field equals the kinetic energy density of the inflowing plasma (from the stellar wind of the companion star).

$$\frac{B(r)^2}{8\pi} = \frac{1}{2}\rho(r)v^2(r)$$

where $v(r)$ and $\rho(r)$ are the velocity and the density of the infalling plasma at a distance r from the neutron star. $v(r)$ can be approximated to

$$v(r) = \sqrt{\frac{2GM}{r}}$$

which is the escape velocity at the radius r. Then, with the continuity equation, one can write the mass loss rate $\dot{M} = 4\pi r^2 \rho(r) v(r)$. With the assumption that the neutron star magnetic field is a dipole, the expression for the Alfvén radius can be derived as

$$R_A = \left(\frac{B_s^2 R_s^6}{\dot{M}\sqrt{2GM}} \right)^{2/7}$$

where B_s and R_s are the surface dipole magnetic field strength and the radius of the neutron star, M is the mass of the neutron star, and G is the gravitational constant.

Corotation radius of the neutron star is the radius at which the Keplerian orbital period of a test particle revolving around the neutron star equals the rotation period of the neutron star. This is given by the expression

$$R_c = \left(\frac{G\,M}{\omega^2} \right)^{1/3}$$

The spin evolution of neutron stars in binary systems go through three distinct phases. A brief summary of these three phases are given below.

Phase I: $r_{lc} < R_A$

This is the limit where the light cylinder radius (r_{lc}) is less than the Alfvén radius (R_A). In this phase the magnetospheric boundary is outside the light cylinder. The matter from the companion (the accreting matter) can not enter the light cylinder and is accelerated to relativistic energies by the low-frequency waves

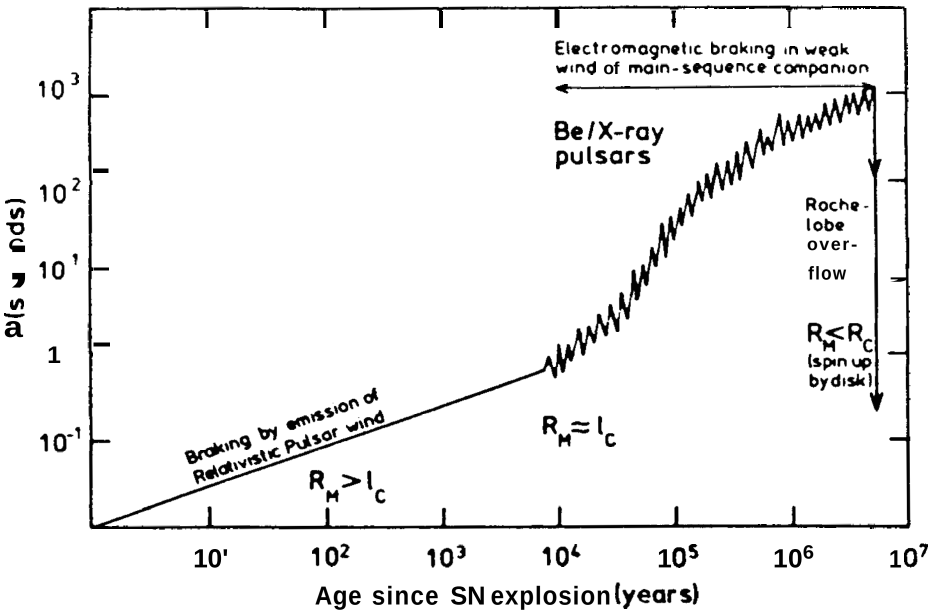

Figure A.l: (taken from van den Heuvel & Habets 1985) A schematic diagram of the evolution of a magnetised neutron star in **a** massive binary system. R_A, R_c and r_{lc} are the Alfvén radius, corotation radius and the light cylinder radius, respectively.

emitted by the pulsar. The braking time scale for rotation is of the order of 1000 to 10^5 years (see figure A.1).

Phase II: $R_c < R_A < r_{lc}$

Here, R_c is the corotation radius $(GM/\omega^2)^{1/3}$ of the pulsar, where the matter corotating with the pulsar just moves with Keplerian velocities. For radius $r < R_c$, corotating matter falls to the neutron star; for $r > R_c$, the corotating matter is accelerated outwards as its centrifugal acceleration exceeds the gravitational acceleration excerted by the central pulsar. When $R_c < R_A$ accretion is not possible, since matter couples to the magnetic field lines at $r = R_A$ and is forced to corotate at radii smaller than R_A. The matter is believed to excert a strong braking torque in this phase near the magnetospheric boundary, though the exact braking mechanism is not clearly understood yet.

Phase III: $R_A > R_c$

This condition is easily satisfied when the companion star is overflowing its Roche-lobe. During this phase the neutron star gets spun-up to short rotation periods, and it may begin to function again as a pulsar. This phenomenon is called recy-*cling.* The final rotation period of the neutron stars is determined by parameters like the dipole moment of the neutron star magnetic field, mass accretion rate etc. The equilibrium period line is given by the expression,

$$P_{eq} = 2.4 \text{ ms } B_9^{6/7} M^{-5/7} \left(\frac{\dot{M}}{\dot{M}_{\text{edd}}} \right)^{-3/7} R_6^{16/7}$$

where B_9, M and R_6 are the magnetic field strength of the neutron star in units of 10^9G, its mass in solar units, and its radius in units of 10^6 cm, respectively. $\dot{M}_{\text{edd}}$ is the Eddington accretion rate. For neutron stars (with assumed mass of

$1.4M_\odot$ the Eddington accretion rate is about $10^{-8}M_\odot\mathrm{yr}^{-1}$. Figure 2.3 of Chapter 1 gives this equilibrium period line in a $B-P$ graph.

It must be also noted here that when some mass is transferred from one star to the other, or when some mass is lost from the system, the orbital parameters change. In massive binaries when the secondary overflows its Roche-lobe the system gets into a common envelope phase. Due to the friction between the neutron star and the surrounding envelope a lot of heat is produced (of course, at the cost of the orbital energy), which is used to expel out the surrounding matter from the system. For systems with orbital periods less than about a year the orbital energy may not be enough to expel out the whole of the envelope. Therefore, the neutron star spirals down to the core of the companion. The system may leave behind only one neutron star at the end of this phase, and this neutron star may be a recycled one. However, in systems with orbital periods greater than a year the binary may survive after ejecting out the envelope, consisting of a neutron star and the core of the companion. If the core of the companion is massive enough, then it will explode as a supernova at the end of its evolution, leaving behind a neutron star. If the binary survives during the supernova explosion, then it leaves behind a double neutron star system (like PSR B1913+16 and B1534+12) with one recycled pulsar and one ordinary pulsar (HMBP systems), and if it does not survive, then the system gives two solitary pulsars.

Final periods and fields

The solitary pulsars seem to have magnetic field strengths of about $\sim 10^{12}$G. The recent statistical studies have shown that the magnetic fields of solitary pulsars do not decay significantly in their lifetimes (Bhattacharya 1992). However. there are strong evidences to suspect that the magnetic fields of pulsars in binary systems decay. There are a few models to explain the field decay in binary systems. For example, the model due to Srinivasan et al. (1990) relates the amount of field decay to the amount by which the neutron star gets spun-down during Phase II, described above. For binaries with massive companions the neutron star is not spun-down to very long periods since the evolutionary time scales are short. Therefore the magnetic field is not expected to decay significantly; whereas in low mass binaries, where the mass of the binary companion is $\lesssim 1 M_{\odot}$, since the slow-down phase continues for Billions of years, and since the neutron star is spun-down to rotation periods of thousands of seconds, the field decay is quite significant. Figure **2.3** of Chapter **1** gives these different possibilities. Neutron stars from'massive binaries do not undergo significant field decay, and they are spun-up to periods $P_{eq} \sim 1$sec. and deposited in the main population of pulsars itself (track 1). Neutron stars from very low mass binaries, after undergoing significant field decay, get spun-up to very short rotational periods of the order of a few milliseconds (LMBP systems). The intermediate mass binaries give rise to neutron stars with configurations in between these two populations (track **2).**