Chapter 3

magnetic fields of neutron stars : a general introduction

3.1 overview

In the cosmic scheme of things the major players behind most of the interesting phenomena are rotation and magnetic field. A unique combination of very rapid rotation and a large magnetic field is what makes a neutron star act as a pulsar. The rotation period of a pulsar can be as small as 1.6 millisecond (Backer et al., 1982), and even the smallest field observed in pulsars could be about three orders of magnitude higher than the maximum field so far achieved in terrestrial laboratories. The typical values of magnetic field in pulsars range from $\sim 10^8$ Gauss to $10^{13.5}$ Gauss. It is the ultra-compact nature of a neutron star that allows the extremes in both the spin rate and the magnetic field strength. Because of the compactness it can support a fast rotation against the centrifugal forces and at close to nuclear densities even such high fields do not affect the state of the matter significantly because the energy associated with the magnetic field is insignificant compared to the other relevant energy scales (Shapiro & Teukolsky 1983).

It should also be noted that the neutron star material is something like the ultimate high-T_c super-conductor. Even though the temperature in the interior of a newly born neutron star could be about 10^9K, it is still small compared to the superconducting transition temperature, believed to be a few times 10^9K. Hence, the material inside a neutron star quickly settles into a superconducting state soon after its birth in a supernova explosion (Alpar 1991, Pines 1991). We shall see later that this plays an important role in shaping the magnetic history of the star.

Unfortunately, there is as yet no satisfactory theory for either the generation of the neutron star magnetic field or its subsequent evolution (Bhattacharya 1995a, 1996b). There is a major uncertainty even about the possible location of the field in the interior of the star. This question is in fact related to the problem of the epoch and mechanism of field generation, as we shall see in the next section (Srinivasan 1995). It is obvious that, under these circumstances, there can be no consensus regarding the theory of field evolution as any such scenario will have to ultimately depend on the nature of the underlying structure and location of the current loops that support the observable field.

Nevertheless, a compilation of current observational facts provides us with the nature of the questions that need to be looked into. They also give an indication of the range of possible answers. These observational facts strongly suggest that the field evolution is intricately related to the binary history of a neutron star. In the rest of this chapter we shall discuss the observational status and the current theoretical attempts to understand the generation and the subsequent evolution of the magnetic field in neutron stars. This provides the background for the problems addressed by us in chapters [4], [5], [6] and [7].

3.2 origin

There are two main possibilities regarding the generation of the magnetic field in neutron stars (for a review see Bhattacharya & Srinivasan 1995, Srinivasan 1995 and references therein). The field can either be a fossil remnant from the progenitor star, or be generated after the formation of the neutron star. Uncertainties surround both the scenarios and observations are yet to be able to distinguish between the two. This has led to a large variety of field evolution models that we shall discuss in section [3.3].

3.2.1 fossil field

Originally suggested by Woltjer (1964) and Ginzburg (1964) long before the discovery of pulsars, the idea of the fossil field is considered to be the most promising. The magnetic field existing in the core of the progenitor star gets enhanced when the core collapses in a supernova, conserving the magnetic flux. Flux conservation demands an increase in the field strength by a factor $(R_{\rm progenitor}/R_{\rm NS})^2$ which is of the order of 10^{10}. This can, depending on fields in the cores of the progenitor stars, produce fields as large as $10^{14} - 10^{16}$ Gauss. The field observed on neutron stars is mainly the dipole component of the surface field. It is possible that the subsurface/interior field is much higher than this value.

In the core of a neutron star, the proton fraction is small (a maximum of 10% when presence of exotic states like Kaon condensates is considered (Pethick & Ravenhall 1992)). Nevertheless, the protons are believed to exist in a superconducting state. Calculations indicate that this is a type II superconductor with a lower critical field in excess of 10^{15} Gauss. Evidently, the observed field values fall far short of this critical field and one expects a complete flux expulsion in accordance with Meissner effect. But unlike in a laboratory situation the flux expulsion encounters a problem because the electrons coexisting with the protons do not form a condensate state. The time scale in which the flux can be expelled is therefore dictated by ohmic diffusion through the electron component. Even at extremely high temperatures at the time of the birth of the neutron star this ohmic diffusion time turns out to be larger than 10^8 years. Hence, superconducting transition occurs with the field embedded in (Ginzburg and Kirzhnits 1964). The magnetic field is carried in quantized flux tubes called Abrikosov fluxoids, each fluxoid carrying a quantum of magnetic flux, $\phi = \frac{hc}{2e} = 2 \times 10^{-7}$ G-cm^2. Therefore, some 10^{31} such fluxoids would be present in a neutron star with a typical field of 10^{12} Gauss.

One major problem with the concept of a fossil field is that strong surface fields are not observed in massive stars except in some dynamically peculiar ones. Ruderman & Sutherland (1973) suggest that the so-called fossil magnetic field may not be a relic of the main sequence phase of the star but can be generated in the core during the turbulent Carbon-burning phase. The strong field can therefore be hidden in the core, but given the short duration of evolution during and after the Carbon-burning phase, it is unclear whether this field can organize itself into large-scale poloidal components. A new input into the physics of the fossil field has come from the refined many-body calculations performed recently on the behaviour of the core superconductor. These new results hint that the core superconductor is likely to be of type I (Ainsworth, Wambach & Pines 1989). This would mean a completely different structure for the field and would require a redressal of some of the existing theories of field evolution involving flux expulsion associated with the spin-down of pulsars (discussed in subsection [3.3.1]).

3.2.2 post-formation field generation

Almost all of the existing field evolution models ultimately depend on ohmic dissipation of the currents in the crust of a neutron star, where the electrical conductivity, though very large by any terrestrial standard, is still finite (in contrast to the superconducting interior which can be thought to have an infinite conductivity). Therefore, any model that makes generation of currents in the crust possible is a very attractive proposition. We have mentioned earlier that the interior of a neutron star quickly settles into a superconducting state immediately after birth, as soon as its temperature falls below a few times 10^9 K. This transition happens within a day or so after the formation of the star, and the observed field values are much smaller than the critical field of this superconductor. Hence, any field that is generated after the formation of the star has to be embedded in the region in which the protons are in a normal'state. So post-formation generation mechanisms give rise to fields that are completely crustal and in particular confined to densities below neutron drip (Blandford et al. 1983, Romani 1990, Urpin & Muslimov 1992).

Following an idea originally proposed by Yakovlev & Urpin (1980), Blandford, Hernquist & Applegate (1983) worked out a possible mechanism for field generation. They suggested that magnetic field arises as a consequence of thermal effects occurring in the outer crust in the early phases of the thermal evolution of the star immediately after it is formed. The investigation is confined to the degenerate surface layers of non-rotating neutron stars, assuming that the plasma is in hydrostatic equilibrium. The density range considered is $\sim 10^7 - 10^{11} \mathrm{g\,cm^{-3}}$, where the mechanism for field enhancement functions most effectively.

The field can grow either in the liquid phase and then be convected into the solid regions, or it could grow in the solid crust itself. In the solid, the heat flux is carried by the degenerate electrons giving rise to thermoelectric instabilities that in turn make the horizontal components of the magnetic field grow exponentially with time. The field grows with a time-scale of $\sim 10^5$ years. Such instability can also develop if the liquid phase that lies above the solid contains a horizontal magnetic field. The coexistence of a heat flux and a seed magnetic field, in excess of 10^8 Gauss, in the liquid will cause the fluid to circulate which may lead to effective dynamo action. If that is the case then the field will grow rapidly, with a time scale of about a 10^2 years, and supply the flux to the solid. Either of these two instabilities will soon saturate to produce a field strength of $\sim 10^{12}$ Gauss, where the instabilities become non-linear. Further growth will be prevented when either the magnetic stress exceeds the lattice yield stress or the temperature perturbations become non-linear, both of which happen at a subsurface field of $\sim 10^{14}$ Gauss. The corresponding surface field is $\sim 10^{12}$ Gauss.

The above mentioned mechanism is most effective for large temperature gradients. And the instabilities grow only if the ohmic diffusion time-scales are such that the increase in the field is not dissipated away at a faster rate than the growth. Unfortunately, for that to happen in the solid the conductivity should be at least a factor of three higher than the present estimates. Or if the surface layer is made of helium the instabilities can grow to large values. It must be noted here that the mechanism of field-growth mentioned here generates small loops only. The growth time-scales in the solid is too long which makes the field generation in the liquid layers the only viable option. The most effective way is to generate the flux in the liquid and then quickly anchor it in the solid crust either by advection of the flux or by freezing the liquid layer itself due to cooling of the star.

This work was extended for the case of rotating neutron stars by Urpin, Levshakov & Yakovlev (1986). They studied the thermo-magnetic instability in less deep and hence less dense liquid layers. In non-rotating neutron stars, the smearing of the instability due to hydrodynamic motions is an impediment to fast growth of field in the liquid. Fast rotations ($P \lesssim 1$ s), however suppresses these hydrodynamic motions by Coriolis force. This mechanism is most effective when the electron temperature is $\gtrsim 3 \times 10^6$ K and leads to the growth of an azimuthally symmetric toroidal magnetic field. Typical time-scale of field growth is of the order of a year and the typical horizontal wavelength is about 100 metres. The field is created at a depth of about 50 metres at density of the order of $10^7 \, \mathrm{g\,cm^{-3}}$.

The post-formation field generation mechanism is besieged by a number of problems, which were recognized by the early workers themselves. First and foremost is the fact that this mechanism is capable of generating only toroidal modes. And the scale-size of the field is confined to the melting depth of the crust which is of the order of hundred meters. In a series of papers, Geppert & Wiebicke (1991, 1995) and Wiebicke & Geppert (1991, 1992, 1995, 1996) have investigated the problem of field generation in the crust of neutron stars in detail. Their work reiterates the fact that the temperature gradient driven thermo-magnetic instability can only give rise to toroidal field, albeit strong. They have shown that it is possible to generate toroidal modes of very high polarities ($n \sim 1000$) in a short time ($\sim$ years) and that the magnitude of such field could also be quite large. But how such field could get restructured to a generate large-scale polar fields, the only kind that is actually observed in neutron stars, is still a point of much speculation.

As yet, there is no observational evidence for distinguishing one kind of field generation mechanism from the other. The only hope lies in the nature of the evolution of the field itself. Since the generation mechanism decides the question of underlying current structure, one expects that the predicted field evolution would be different for these two cases, and observations would be able to probe this difference. In chapter [8] we shall make some comments about the conclusions we have drawn regarding this problem from the nature of the field evolution and the evolutionary link between the population of normal pulsars and their millisecond counterparts.

3.3 evolution

The evolution of neutron star magnetic field has been of abiding interest both because it is a challenging problem in itself and also due to the fact that many other aspects of pulsar physics crucially depend on the nature of field evolution. Even though over the years a somewhat coherent picture has emerged, some of the key issues still remain unresolved. In particular, the generation of millisecond pulsars and their evolutionary link to the population of normal pulsars through a processing in binaries has become one of the major challenges that faces the researchers at present. There have been several excellent reviews detailing the current worries and the status of the theoretical endeavors (Bhattacharya 1995a, 1996b, Bhattacharya & Srinivasan 1995, Ruderman 1995). In this chapter, we shall recapitulate, in brief, the present situation, the observational indications and the theoretical models.

From observational facts and from the recent statistical analyses made on the pulsar population, the following conclusions can be drawn regarding the field evolution.

- Isolated pulsars with high magnetic fields ($\sim 10^{11} - 10^{13}$ G) do not undergo any significant field decay during their lifetimes (Bhattacharya et al. 1992; Wakatsuki et al. 1992, Lorimer 1994, Hartman et al. 1997).
- The fact that binary pulsars as well as millisecond and globular cluster pulsars which almost always have a history of being a member of a binary, possess much

lower field strengths, suggests that significant field decay occurs only as a result of the interaction of a neutron star with its binary companion (Bailes 1989).

- The age determination of pulsars with white dwarf companions show that most of the millisecond pulsars are extremely long-lived. The old age ($\sim 10^9$ years) of the low-field pulsars implies that their field is stable over long time-scales. That is after the recycling process in the binaries is over, the field does not undergo any further decay (Bhattacharya & Srinivasan 1986; Kulkarni 1986; van den Heuvel, van Paradijs & Taam 1986; Verbunt et al. 1990).

- The evolutionary link between millisecond pulsars and low-mass X-ray binaries seem to be borne out both from binary evolution models and from the comparative study of the kinematics of these two populations. To spin a neutron star up to millisecond periods at least an amount $\sim 0.1 M_\odot$ needs to be accreted. Such huge amount of mass transfer is possible only in low mass X-ray binaries, where the duration of mass transfer could be as long as 10^9 years. And even though the present estimates for the birthrate of low mass X-ray binaries fall short of that of the millisecond pulsars (Kulkarni & Narayan 1988; Lorimer 1995), kinematic studies (Cordes et al. 1990; Wolszcan 1994; Nice & Taylor 1995; Nicastro & Johnston 1995; Bhattacharya 1996a, Ramachandran & Bhattacharya 1997; Cordes & Chernoff 1997) indicate that the two populations are most likely to be related as both have very similar kinematic properties.

3.3.1 models of field evolution

In order to understand the above facts, attempts have been made to relate the field decay to the star's binary history. There are two classes of models which have been explored in this context one that relates the magnetic field evolution to the spin evolution of the star and the other attributing the field evolution to direct effects of mass accretion (see Ruderman (1995) and Bhattacharya (1995a) for detailed reviews). It should also be mentioned here that almost all the models are built upon two themes, namely, a large scale macroscopic restructuring of the fields in the interior of the star (for example, the spin-down and expulsion of flux from the superfluid core of the star) and a microscopic mechanism (like ohmic dissipation of the currents in the crust) acting to actually kill the underlying currents supporting the observable field. The different class of models usually assume different kind of initial field configuration. Models depending on spin-down assume a core-flux supported by proton superconductor flux tubes. Whereas, models invoking ohmic dissipation usually assume an initial crustal configuration. It should be noted here that a whole host of models exist that discuss field evolution of isolated neutron stars (for a list of such models see Bhattacharya 1996b). But we shall confine our discussion to accretion-induced models alone.

The former class of models involves the inter-pinning of the Abrikosov fluxoids (of the superconducting protons) and the Onsager-Feynman vortices (of the superfluid neutrons) in the core (Srinivasan et al. 1990; Ruderman 1991a). This class also includes the models involving the plate tectonics of the neutron star crust (Ruderman 1991b, 1991c). Srinivasan et al. (1990) pointed out that neutron stars interacting with the companion's wind would experience a major spin-down, causing the superconducting core to expel the magnetic flux, which would then undergo ohmic decay in the crust. This coupled evolution of spin and magnetic field has been modeled both for the cases of wide low mass X-ray binaries (Jahan Miri & Bhattacharya 1994) and high mass X-ray binaries (Jahan Miri 1996). They have assumed ohmic decay of the expelled field in the crust with a constant decay constant of $10^8 - 10^9$ years. An investigation of the nature of the ohmic decay of such expelled field in an accretion heated crust has also been attempted (Bhattacharya & Datta 1996), and shows that the final field values could be quite consistent with those observed for millisecond pulsars. Ruderman (1991b, 1991c) on the other hand, suggests a coupling between the spin and the magnetic evolution of the star via crustal plate tectonics - torques acting on the star cause crustal plates, and the magnetic poles anchored in them, to migrate, resulting in major changes of the effective dipole moment.

The other class of models attribute the field decay to direct effects of accretion. The work done in this thesis comes under this category of models. Previous work by Bisnovatyi-Kogan & Komberg (1974) and Taam & van den Heuvel (1986) have suggested that accreted matter might screen the pre-existing field. Computations by Romani (1990) indicate that hydrodynamic flows may bury the pre-existing field reducing the strength at the surface. We shall see in chapter[4] that strong Rayleigh-Taylor instabilities prevent such hydrodynamic flows to create horizontal components of the field at the expense of the dipolar component, and therefore such a screening mechanism does not provide for a viable scenario of field evolution in an accreting neutron star. The mechanism of ohmic decay, being unique to the crustal currents, is also used in models where spin-down is invoked for flux expulsion, for a subsequent dissipation of such flux in the crust (Jahan Miri & Bhattacharya 1994, Bhattacharya & Datta 1996).

The most important microscopic mechanism invoked for accretion-induced field decay is that of fast ohmic decay. In an accretion-heated crust the decay takes place principally as a result of rapid dissipation of currents due to the decrease in the electrical conductivity and hence a reduction in the ohmic dissipation time-scale (Geppert & Urpin (1994), Urpin & Geppert (1995), Urpin & Geppert (1996), Konar & Bhattacharya (1997)). The crustal field undergoes ohmic diffusion due to the finite electrical conductivity of the crustal lattice, but the time-scale of such decay is very long under ordinary conditions (Sang & Chanmugam 1987; Urpin & Muslimov 1992). The situation changes significantly when accretion is turned on. The heating of the crust reduces the electrical conductivity by several orders of magnitude, thereby reducing the ohmic decay time-scale.

There is also an additional effect that acts towards stabilizing the field. As the mass increases, a neutron star becomes more and more compact and the mass of the crust actually decreases by a small amount. So the newly accreted material forms the crust and the original crustal material gets continually assimilated into the superconducting core below. The original current carrying layers are thus pushed into deeper and more dense regions as accretion proceeds. The higher conductivity of the denser regions would progressively slow down the decay, till the current loops are completely inside the superconducting region where any further decay is prevented (Konar & Bhattacharya 1997).

3.3.2 the millisecond pulsar question

A summary of the current status of the important question of generation of millisecond pulsars has recently been presented by Bhattacharya (1996a) in the proceedings of the IAU colloquium 160. The debate that followed the presentation (recorded in the same proceedings) gives a fair indication as to how bad the situation is. Here we shall'just record the facts relevant to the field evolution, ignoring such questions that are associated with binary evolution and other factors.

In the fifteen years following the discovery of the first millisecond pulsar more than fifty pulsars have been discovered which belong to this particular category. In an attempt to understand the origin of millisecond pulsars, it was suggested that these are ***recycled*** pulsars, pulsars that have evolved to the characteristic magnetic field and spin-period by virtue of their binary history.

The major problem regarding the generation of millisecond pulsars in the binaries is the question of progenitors. What kind of binaries would give rise to millisecond pulsars? We have already mentioned that there is a problem of birthrate mismatch if one assumes that all the millisecond pulsars come from low mass X-ray binaries we normally observe. There is no satisfactory explanation for the origin of isolated millisecond pulsars either. Any model for field evolution has to be consistent with the nature of binary evolution that produced these objects. The field evolution scheme must also provide for a limiting minimum field strength—the so-called 'flooring' seen at $\sim 10^8$ G should arise out of the evolution itself. Except for a few of the models (Romani 1995, Jahan Miri & Bhattacharya 1994, Konar & Bhattacharya 1997) an explanation for this has not been attempted so far. And the models of field evolution must also match the spin-evolution of the neutron star in the binaries. Since there is no consensus on either the internal field structure or the evolution of the field, one important check on such models would be to look for a consistency of the field evolution models with that of the binary evolution scenario. In chapter[6] we attempt to look for such a consistency in general and from the point of view of millisecond pulsar generation in particular.

3.4 general introduction to accretion-induced field evolution

In the previous section we have seen that observational facts and statistical analyses of the existing data clearly relate the evolution of magnetic field of neutron stars with binary interaction. In the next chapter we shall address three different aspects of such binary-induced field evolution. We investigate the underlying physical mechanisms that take place in the interior of the star, changing the currents that are flowing to maintain the observable field. We would interest ourselves with a neutron star that is in active interaction with its binary companion. This companion could be a main sequence star affecting the neutron star through its wind. Alternatively, it could be a giant or a super giant, in which case heavy mass transfer may take place through Roche-lobe overflow (Bhattacharya & van den Heuvel 1991, King et al. 1995). In either of these cases, there are some basic changes that the neutron star undergoes. These changes in various physical parameters are responsible for a change in the magnetic field. So, here is a brief resume of how the process of accretion affects a neutron star.

- The process of accretion mainly changes three parameters of the star, the total mass, the total angular momentum and the total energy. The change in energy comes about when the potential energy of the accreting particles get converted into random kinetic energy of the system and is then manifested as an increase in temperature.

- Models of field evolution have been proposed using the changes in each of these quantities, namely relating the change in the observable field SB to SM (change in mass), δJ (change in angular momentum) and ST (change in temperature). In this connection it should be mentioned that earlier attempts to show a proportionality between SB and SM have recently been challenged Wijers (1996).

- The change in angular momentum has been exploited in models which assume an initial field confined to the superconducting core of the star. In particular, a rapid slow-down ($SJ \ll 0$) experienced by the neutron star during the 'propeller phase' causes the core field to be expelled to the crust, where it could undergo subsequent ohmic dissipation.

- Ultimately most of the accretion-related models depend on the ohmic dissipation of currents in the crust for a permanent decrease in the field. The rapidity of such dissipation depends on a decrease of the electrical conductivity, caused by an increase in the temperature ($ST \geq 0$).

- Another important factor influencing the field evolution is the hydrodynamic mass motion in the star. Near the surface the material flow is lateral, moving from polar (magnetic) to equatorial region. But interchange instabilities appear to prevent the screening of the field due to such hydrodynamic motions (chapter [4] of this thesis).

- Deep inside the star the material movement is a simple radial inflow, due to the compression of the star as a result of an increase in the total mass. The material movement is, evidently, a direct effect of the change in mass (SM), with the actual flow dynamics determined by the rate of such change, i.e., the rate of mass accretion, M, on the star. This radially inward motion in the deeper layers helps to advect crustal currents to more dense layers, ultimately stabilizing the field in such cases.

From the above discussion, we see that processes of both macroscopic and microscopic nature are active in the interior of an accreting neutron star. The change in angular momentum induces a large scale flux movement whereas a change in total mass gives rise to large scale material movement. Both these cause macroscopic restructuring of the current pattern. On the other hand, a change in temperature induces the microscopic process of ohmic dissipation, by which the energy stored in the large scale field is transferred to the random kinetic energy of the particles. In the following sections, we shall see how these two mechanisms complement each other in the problems addressed by us.

Chapter 4

effect of diamagnetic screening

4.1 introduction

First proposed by Bisnovatyi-Kogan & Komberg (1974), the idea of a possible screening of the magnetic field of a neutron star by accreting material onto it, has been a recurrent theme in the field-evolution scenario. This suggestion was substantiated by the work of Taam & van den Heuvel (1980) in which they indicated that the accreted matter, which is completely ionized plasma and hence diamagnetic in nature, might screen the pre-existing field reducing the strength of the surface field. Later Blandford, De Campli & Konigl (1979) proposed a possible mechanism for such a screening. They suggested that the material accreting onto the poles of a magnetized neutron star will be confined by the strong magnetic stresses near the surface of the star. At low accretion rates the material sinks below the stellar surface until the hydrostatic pressure of the stellar material is as large as the magnetic pressure. The plasma then flows sideways giving rise to horizontal components of the magnetic field. The creation of this horizontal component comes at the expense of the vertical field and may result in a decrease in the observed field strength. This work, however, did not provide any quantitative details of the mechanism.

Later it was shown by Woosley & Wallace (1982) and Hameury, Bonazzola, Heyvaerts & Lasota (1983) that the accretion column is actually like a small mountain on the polar cap rather than being subsurface. The accreting material is mostly ionized hydrogen and hydrogen has a smaller mean molecular weight per electron, μ_e, than the iron crust of the star. Since most of the pressure in the accreted layer comes from the electrons, hydrogen tends to float over the underlying iron layer - if no material displacement perpendicular to the magnetic field is possible. A hydrogen mountain then forms at the surface of the neutron star. The height of this mountain depends on the density at which the transmutation of hydrogen (by electron capture on protons) takes place. When the material pressure in this accretion column becomes much larger than the magnetic pressure then the material starts flowing sideways giving rise to horizontal field components reducing the external dipolar field strength.

The first quantitative calculations of diamagnetic screening of neutron star magnetic field were performed by Romani (1990). It was shown in this work that hydrodynamic flows in the surface layers may bury the field to deeper layers effectively reducing the surface strength. In a later article (Romani, 1995) in continuation to the earlier work, various time-scales, relevant for an effective screening of the surface field, were estimated. It was also shown here how the depth and density at which the field might get buried depend on the strength of the initial surface field.

In the present work, we investigate the effectiveness of diamagnetic screening as a possible mechanism for a permanent reduction in the surface field strength of the neutron stars. In particular, we shall try to answer the following questions :

1. Are diffusive time-scales, in the layers where the field is expected to be buried, long enough to allow screening to be an effective mechanism for long-term field reduction ?

2. Is it at all possible to bury the field or create horizontal components at the expense of the vertical one against Rayleigh-Taylor overturn instability ?

Recently, calculations of screening have been performed assuming the accreting material to be ferromagnetic (Zhang, Wu & Yang 1994, Zhang 1997). Such an assumption of extremely large magnetic permeability makes it possible to reduce the surface field by about four orders of magnitude. In our work though, we regard the accreting material to be completely ionized plasma (mainly hydrogen) and therefore to be diamagnetic in nature.

The layout of the chapter is in the following form. In section [4.2] we have discussed the nature of the material flow in an accreting neutron star. In section [4.3] we discuss the actual mechanism of the diamagnetic screening and the various relevant time-scales. And finally in section [4.4] we present our conclusions.

4.2 accretion and material flow

For a neutron star with a strong magnetic field, the flow of the accreting material outside the star is completely determined by the field. The material flows in along open field lines and hits the surface within the polar cap region. The area of this polar cap region is determined by the rate of accretion and the strength of the magnetic field (Shapiro & Teukolsky, 1983). In all our subsequent discussions we shall assume a situation where the magnetic axis and the rotation axis are aligned. Even though that is hardly ever the case in reality, this assumption does not affect the general conclusions much.

The polar cap area, region constrained by open field lines on the surface of the star, is given by,

$$A_P = 2\pi R_s^2[1 - \cos\theta], \tag{4.1}$$

where, R, is the radius of the star and θ is the angle the last open field line makes with magnetic axis. This limiting field line is the one which goes through the Alfvén radius, R_A, in the equatorial plane. Therefore, using the equation for the dipole field lines one finds $\sin 0 = \sqrt{\frac{R_s}{R_A}}$, so that the polar cap area is given by,

$$A_P = 2\pi R_s^2\left[1 - \sqrt{1 - \frac{R_s}{R_A}}\right]. \tag{4.2}$$

In the limit $R_A >> R_s$ (which is a reasonable approximation for the range of field strength and accretion rate we consider), the above expression reduces to the following simple form :

$$A_P = \pi\frac{R_s^3}{R_A}. \tag{4.3}$$

The Alfvén radius of an accreting system is determined by the condition that at this radius the ram pressure of the in-falling material equals the pressure of the magnetic field. The ram pressure at the Alfvén radius is :

$$P_{\rm ram} = \frac{1}{2}\rho(R_A)V^2(R_A), \tag{4.4}$$

where $\rho(R_A)$ is the density and $V(R_A)$ is the velocity of the accreting material at Alfvén radius. In an accretion disc material rotates with Keplerian velocity at every radius. Therefore, the velocity of the material at Alfvén radius is given by,

$$V(R_A) = (\frac{GM}{R_A})^{1/2}. \tag{4.5}$$

The density of the accreting material at Alfvkn radius is :

$$\rho(R_A) = \frac{\dot{M}}{4\pi V(R_A) R_A^2}, \tag{4.6}$$

where, the dependence on the rate of mass accretion is quite obvious. As the field is dipolar in nature, the field strength at the Alfvkn radius is,

$$B(R_A) = (\frac{R_s}{R_A})^3 B_s, \tag{4.7}$$

where, B_s stands for the strength of the field at the surface of the neutron star. Equating the ram pressure and the magnetic pressure, one obtains the following expressions for the Alfvkn radius and the polar cap area respectively,

$$R_A = (2GM)^{-1/7} R_s^{12/7} B_s^{4/7} \dot{M}^{-2/7}, \tag{4.8}$$

$$A_P = \pi (2G)^{1/7} M^{1/7} R_s^{9/7} B_s^{-4/7} \dot{M}^{2/7}. \tag{4.9}$$

The density profile within the accretion column is described by an 'atmosphere' solution (Bildsten & Cutler, 1995). The pressure in the column, due to the relativistic degenerate electrons and the ions, is given by

$$P = \frac{1}{4} n_e m_e c^2 x + n_I k_B T, \tag{4.10}$$

where $n_e (n_I)$ is the number density of electrons (ions) and m_e is the electron mass. x is the relativity parameter defined as

$$\begin{aligned} x &= \frac{p_F}{m_e c} \\ &= 1.008 (\frac{Z \rho_6}{A})^{1/3}, \end{aligned} \tag{4.11}$$

where p_F is the Fermi momentum of the electrons, (Z, A) correspond to the atomic number and mass number of the dominant ion species and ρ_6 is density in units of $10^6 \mathrm{g\,cm^{-3}}$. The scale height of the accretion column is given by $\mathrm{H} = \frac{P}{\rho g}$, where ρ and P are the density and pressure respectively at the base of the accretion column and g is the acceleration due to gravity at the surface of the state. Using the above equation of state the scale height, for a 1.4 $M_\odot$ star, is approximately found to be :

$$H \sim 324\, \rho_6^{1/3} \mathrm{cm}. \tag{4.12}$$

As long as the pressure in the accretion column is much smaller than the magnetic pressure there can not be any material movement across the field lines so the accreted material remains confined to the polar cap. This kind of polar cap accretion causes transverse pressure gradients that are balanced by the curvature of the magnetic field.

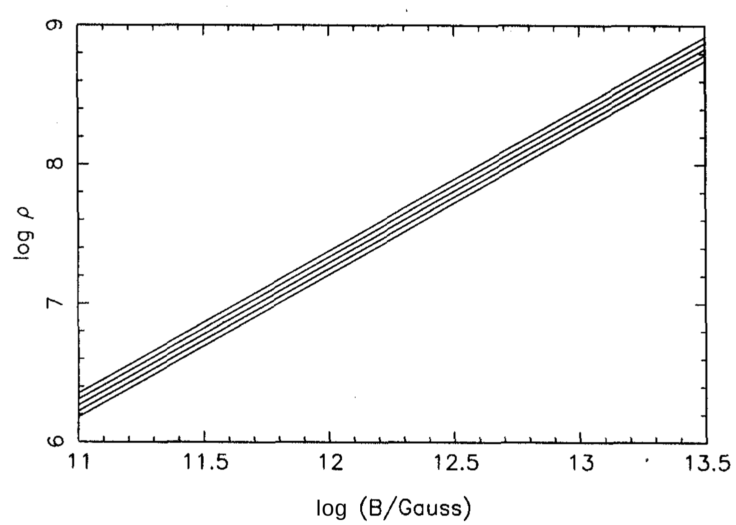

Figure 4.1: Variation of the flow density with the surface fields strength. The different curves (from top to bottom) correspond to M = $10^{-8}, 10^{-8.5}, 10^{-9}, 10^{-9.5}, 10^{-10}$ $M_\odot$ yr^{-1}respectively.

An order of magnitude estimate by Hameury et al. (1983) shows that in order to create a significant distortion of the field lines due to the overpressure of the accretion column the condition

$$\frac{P_{\rm accretion}}{P_{\rm magnetic}} \gtrsim \frac{R_P}{H} \tag{4.13}$$

should be satisfied. Here, $P_{\rm accretion}$ and $P_{\rm magnetic}$ stand for pressure due to the material in the accretion column and due to the magnetic field respectively. R_P is the radius of the polar cap and H is the density scale height of the accretion column. For a given value of the crustal field and a given rate of accretion, the density at which the material starts flowing sideways, obtained from the condition given by equation [4.13], is :

$$\rho_{\rm flow} = 2.0961 \times 10^8 B_{13}^{36/35} \dot{M}_{-9}^{3/35}, \tag{4.14}$$

where B_{13} is the field strength in units of 10^{13} Gauss and $\dot{M}_{-9}$ is the accretion rate in units of 10^{-9} $M_\odot$ yr^{-1}. This condition is, of course, only valid in absence of any interchange instabilities about which we shall discuss later. In figure [4.1] we plot the variation of the flow density with the field strength for various values of the accretion rate.

4.3 physics of diamagnetic screening

We shall, as has been mentioned above, work under the assumption of a ***flux-frozen*** situation. This basically amounts to assuming an effectively infinite conductivity. Admittedly, even though large, the actual value of conductivity is finite. But the large values of conductivity in crust provide for extremely long diffusive time-scales. In this case, the assumption of 'flux-freezing' is a valid one since the flow time-scales are much smaller compared to this diffusive time-scale (to be discussed in the next section).

Therefore a reduction in the surface field is obtained due to the combined effect of i) flux freezing and ii) hydrodynamic flow of the material. Due to flux-freezing the total flux contained within an amount of material remains constant. And when this material flows and spreads out covering a larger area, the field gets adjusted to keep the flux conserved. Say, the total flux contained within the polar cap at time $t = 0$ is B x A_P. Conservation of flux then implies that we have,

$$\frac{dB}{dt} \times A_P + B \times \frac{dA_P}{dt} = 0. \tag{4.15}$$

Therefore the change in the field strength is related to the change in the area of the original polar cap surface due to material flow during accretion. Assuming that the flow of material is such that at each instant the amount accreted equals the amount flowing out from under the accretion column one obtains the following relation between the change in the polar cap area and the rate of mass accretion M given by,

$$\dot{M} dt = \rho_{\text{flow}} h_{\text{flow}} dA_P \tag{4.16}$$

where, dA_P is the change in the polar cap area in time dt. ρ_{flow} is the density where the flow takes place and h_{flow} is the effective height of the accretion column, which hereafter we take roughly to be equal to the density scale height H. Integrating equation [4.16] above the time variation of the polar cap area is obtained in the following form -

$$A_P(t) = A_P(t = 0) + \frac{\dot{M} t}{\rho_{\text{flow}} H}, \tag{4.17}$$

where, $A_P(t = 0)$ is the original area of the polar cap. Then equation [4.15] implies the following time dependence of the surface field :

$$B(t) = \frac{B_0}{[1 + \frac{t}{\tau_{\text{screen}}}]}, \tag{4.18}$$

where we have defined the screening time-scale by the relation

$$\tau_{\text{screen}} = \frac{\rho_{\text{flow}} H A_P}{\dot{M}} \tag{4.19}$$

This is the time in which the original field reduces to half of its original value. It should be noted here that Shibazaki, Murakami, Shaham & Nomoto (1989) obtained an expression for the decay of magnetic field due to accretion very similar to equation [4.15] from purely phenomenological considerations. Here we have arrived at the above relation from a more physical point of view.

To find the effectiveness of the screening we need to compare this time-scale to the diffusive time-scale over which the field may re-emerge by ohmic diffusion through the overlaying layers. The diffusive time-scale is defined by,

$$\tau_{\text{diff}} = \frac{4\pi\sigma(\rho_{\text{flow}}, T(\dot{M}))H^2}{c^2}, \tag{4.20}$$

where, a is the electrical conductivity which is a function of the density and the crustal temperature. We shall show later that if one ignores interchange instabilities, then the flow time scale is much smaller than diffusion time scale, and therefore field would remain frozen and be dragged. Reconnection will then occur on the equatorial plane and that will bury the field. Before this field can diffuse out more matter will come and spread on top of it, and drive the field deeper. So the eventual re-emergence time scale is set not by the initial depth at which it was buried but the final depth to which it is driven by continuous accretion. This can be very deep and the field can even reach the core and therefore never get out again. But for the moment, we take the burial depth to be equal to the scale height of the density. For the screening time-scale this, of course, is the actual length-scale. For the diffusive case we find the time-scale at the depth where the field gets buried to begin with. We have not considered the case of deep burial mentioned above, since from the subsequent discussion that will prove to be unnecessary.

The point to be noted here is that for the screening mechanism described above the important factor is the anisotropic material flow through the polar cap. The area of polar cap increases with increasing rate of accretion. Hence, for low rates of accretion, the flow is maximally anisotropic. For high values of accretion rate, the polar cap could cover a large fraction of the stellar surface area. The effect of anisotropic material flow would not be very severe in that case. Also as mentioned earlier, the temperatures are higher for higher values $\dot{M}$, a lower value of conductivity and therefore a smaller diffusive time. Hence, situations with low values of $\dot{M}$ has better chance of screening the field.

In all of the above discussion, the implicit assumption has been that it is possible to create a horizontal component of the magnetic field, at the expense of the vertical component, as a result of material flow in a 'flux-frozen' condition. But this mechanism is not viable against the Rayleigh-Taylor instability. This instability arises due to magnetic buoyancy. As the accreting material slowly builds up in a column, the field lines tend to spread out due to the diamagnetic property of the material. At the base of the accretion column, the field lines are firmly anchored to the solid crust. Therefore material flow at the base of the accretion column stretches the field lines out horizontally. This creates horizontal components of the field giving rise to a screening of the external dipole. We have seen earlier that the pressure of the accretion column is much larger than the pressure of the magnetic field in this region. Under such a condition the field lines re-organize themselves into flux tubes as that is the minimum energy configuration. These flux tubes move upward due to magnetic buoyancy and thereby destroy the horizontal component of the field (Spruit *1983* and references therein).

In a tube in pressure equilibrium with its surroundings the pressure is

$$P_{\rm in} + P_{\rm magnetic} = P_{\rm out}. \tag{4.21}$$

However at the base of the accretion column $\beta \equiv \frac{P_{\rm gas}}{P_{\rm magnetic}} \sim \frac{R_P}{H}$ and the above relation simplifies to

$$P_{\rm in} = (1 - 1/\beta) P_{\rm out}. \tag{4.22}$$

Therefore, the gas pressure within a flux tube is smaller than the gas pressure outside. Then from equation [4.10] we find the following relation between internal and external densities :

$$Pin = (1 - 1/\beta)^{3/4} \rho_{\rm out} \tag{4.23}$$

or

$$\frac{\delta\rho}{Pin} = \frac{3}{4P}, \tag{4.24}$$

where $\delta\rho$ is the density deficit within the flux tubes. This density deficit makes the flux tube move upward under the force of buoyancy. The upward velocity V_F of the flux tubes is given by the equality of the buoyancy force with the aerodynamic drag force per unit area of the flux tube :

$$\frac{1}{2}\rho_{\rm in} V_F^2 = g\, r\, \delta\rho, \tag{4.25}$$

where r is the characteristic radius of the flux tube and g is the acceleration due to gravity. Therefore, the velocity is :

$$V_F = \sqrt{\frac{3gr}{2\beta}} \tag{4.26}$$

and the time-scale required for a flux-tube to traverse the density scale-height is :

$$\tau_{\rm RT} = \frac{H}{V_F}. \tag{4.27}$$

It should be noted that the characteristic scale length of the flux tubes is again that of the density scale-height, and therefore,

$$\tau_{\rm RT} = \sqrt{\frac{2\beta H}{3g}}. \tag{4.28}$$

The time-scale for the reconnection of the field lines, on the other hand, is typically given by

$$\begin{aligned} \tau_{\rm recon} &= \frac{l_{\rm instability}}{V_A}, \text{ if } V_A < V_S \\ &= \frac{l_{\rm instability}}{V_S}, \text{ if } V_S < V_A; \end{aligned} \tag{4.29}$$

where, V_A, V_S are the Alfvkn and the sound speed, respectively. $l_{\rm instability}$ is the length-scale of the instability which is again of the order of the density scale height in the accretion column. Within the accretion column the sound speed is always much greater than the Alfvkn velocity and therefore only the top expressions of the above equations are relevant here. At the flow density, the Alfvén velocity is

$$V_A = \frac{B}{\sqrt{4\pi\rho_{\rm flow}}}. \tag{4.30}$$

Therefore, the expressions for the reconnection time-scale is given by

$$\tau_{\rm recon} = \frac{H\sqrt{4\pi\rho_{\rm flow}}}{B}, \tag{4.31}$$

(4.32)

For the screening to be effective, the total time taken to create a loop of field lines by turn-over instability and to destroy the loop by reconnection has to be much larger than the screening time-scale defined above. So we define an instability time-scale as

$$\tau_{\rm instability} = \tau_{\rm RT} + \tau_{\rm recon} \tag{4.33}$$

which should be compared with $\tau_{\rm screen}$. It must be noted here that we use the 'pressure scale-height' and 'density scale-height' interchangeably in the above discussion since for barotropic (atmospheric) equation of state (as is the case here) a pressure gradient implies the same density gradient.

Therefore the three relevant time-scales of the problem are :

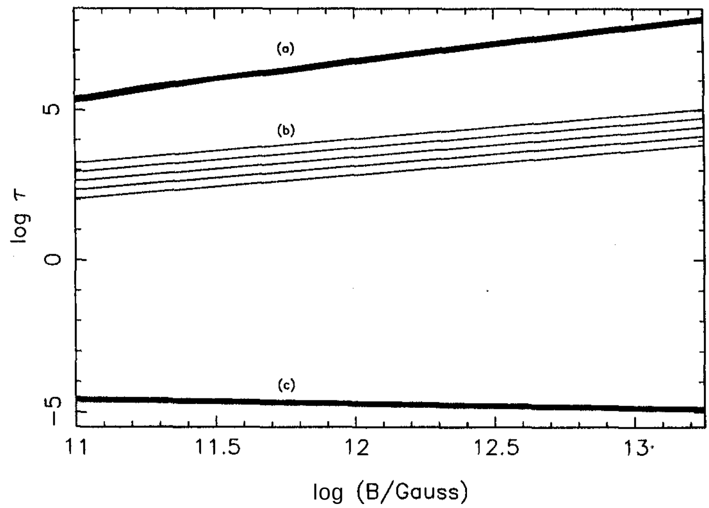

Figure 4.2: Variation of the time-scales with surface fields strength. The (a), (b), and (c) groups of curves correspond to diffusive, screening and instability time-scales respectively. The curves (in each group) correspond to $\dot{M} = 10^{-10}, 10^{-9.5}, 10^{-9}, 10^{-8.5}$ $M_{\odot}$ yr^{-1}respectively, the upper curves being of lower values of accretion.

- screening time-scale - given by equation [4.19];
- diffusive time-scale - given by equation [4.20]; and
- instability time-scale - given by equation [4.33]

In figure[4.2] we plot all the time-scales as functions of surface field strength and the accretion rate.

4.4 discussion and conclusions

Looking at the figures plotted above, obtained for a range of values for the accretion rate and the surface field strength, we find that,

1. The density of flow increases with an increase in the field strength. The larger the field, the deeper and denser it gets buried (see figure [4.1]). The flow density also has a mild positive dependence on the rate of mass accretion. But even for very large values of the surface field strength the flow does not occur at densities beyond $\sim 10^9$ g cm^{-3}. That means the flow always takes place within the liquid

layer. And the earlier contentions of a burial within the solid layer does not really happen.

2. As the screening time-scales are always much smaller than the diffusive time-scales a condition of flux-freezing prevails and material movement indeed should proceed dragging the field lines along.

3. But, since the instability time-scale (sum of the overturn and the re-connection time-scales) is much too smaller than the other two time-scales of the problem, any stretching of field line is quickly restored (over the instability time-scale) and the material effectively flows past the field lines without causing any change.

We find that the time-scale of overturn and reconnection is so much smaller than any other time-scale of the problem that it is not at all possible to create horizontal field components at the cost of vertical ones and effect a screening. Therefore, it is not possible to screen the magnetic field of a neutron star by the accreting material in order to reduce the magnitude of the external dipole observed.